CHƯƠNG 4

GIỚI HẠN

BÀI 2. GIỚI HẠN CỦA HÀM SỐ

A. TÓM TẮT LÝ THUYẾT

Định nghĩa 1 (Giới hạn của hàm số tại một điểm).

Giả sử $(a;b)$ là một khoảng chứa điểm x_0 và f là một hàm số xác định trên tập hợp $(a;b)\setminus\{x_0\}$. Ta nói rằng hàm số f có giới hạn là số thực L khi x dần đến x_0 (hoặc tại điểm x_0) nếu với mọi dãy số (x_n) trong tập hợp $(a;b)\setminus\{x_0\}$ mà $\lim x_n = x_0$ ta đều có $\lim f(x_n) = L$.

Khi đó ta viết $\lim\limits_{x\to x_0} f(x) = L$ hoặc $f(x)\to L$ khi $x\to x_0$.

Định nghĩa 2 (Giới hạn của hàm số tại vô cực).

Giả sử hàm số f xác định trên khoảng $(a;+\infty)$. Ta nói rằng hàm số f có giới hạn là số thực L khi x dần tới $+\infty$ nếu với mọi dãy số (x_n) trong khoảng $(a;+\infty)$ mà $\lim x_n = +\infty$ ta đều có $\lim f(x_n) = L$.

Khi đó ta viết $\lim\limits_{x\to+\infty} f(x) = L$ hoặc $f(x)\to L$ khi $x\to+\infty$.

GIỚI HẠN HỮU HẠN	GIỚI HẠN VÔ CỰC
Giới hạn đặc biệt 1) $\lim\limits_{x\to x_0} x = x_0$. 2) $\lim\limits_{x\to x_0} c = c\ \ (c\in\mathbb{R})$.	**Giới hạn đặc biệt** 1) $\lim\limits_{x\to+\infty} x^k = +\infty$. 2) $\lim\limits_{x\to\pm\infty} \dfrac{c}{x^k} = 0$. 3) $\lim\limits_{x\to 0^-} \dfrac{1}{x} = -\infty$. 4) $\lim\limits_{x\to 0^+} \dfrac{1}{x} = +\infty$. 5) $\lim\limits_{x\to-\infty} x^k = \begin{cases} +\infty \text{ khi } k \,\vdots\, 2\ (k\neq 0) \\ -\infty \text{ khi } k \not\vdots\, 2 \end{cases}$
Định lí Nếu $\lim\limits_{x\to x_0} f(x) = L$ và $\lim\limits_{x\to x_0} g(x) = M$ thì 1) $\lim\limits_{x\to x_0}\left[f(x)\pm g(x)\right] = L\pm M$. 2) $\lim\limits_{x\to x_0}\left[f(x).g(x)\right] = L.M$. 3) $\lim\limits_{x\to x_0}\dfrac{f(x)}{g(x)} = \dfrac{L}{M}$ với $M\neq 0$. Nếu $f(x)\geq 0$ và $\lim\limits_{x\to x_0} f(x) = L$ thì $\lim\limits_{x\to x_0}\lvert f(x)\rvert = \lvert L\rvert$ và $\lim\limits_{x\to x_0}\sqrt{f(x)} = \sqrt{L}$.	**Định lí 1** Nếu $\lim\limits_{x\to x_0} f(x) = L\neq 0$ và $\lim\limits_{x\to x_0} f(x) = \pm\infty$ thì $\lim\limits_{x\to x_0}\left[f(x).g(x)\right] = \begin{cases} +\infty \text{ khi } L.\lim\limits_{x\to x_0} g(x) > 0 \\ -\infty \text{ khi } L.\lim\limits_{x\to x_0} g(x) < 0 \end{cases}$. Nếu $\lim\limits_{x\to x_0} g(x) = 0$ thì $\lim\limits_{x\to x_0}\dfrac{f(x)}{g(x)} = \begin{cases} +\infty \text{ khi } L.g(x) > 0 \\ -\infty \text{ khi } L.g(x) < 0 \end{cases}$.

Giới hạn một bên

$\lim\limits_{x\to x_0} f(x)=L \Leftrightarrow \lim\limits_{x\to x_0^+} f(x)=\lim\limits_{x\to x_0^-} f(x)=L$.

B. DẠNG TOÁN VÀ BÀI TẬP

Dạng 1. Tính giới hạn vô định dạng $\frac{0}{0}$, trong đó tử thức và mẫu thức là các đa thức.

Phương pháp giải:

Khử dạng vô định bằng cách phân tích thành tích bằng cách chia Hooc – nơ (đầu rơi, nhân tới, cộng chéo), rồi sau đó đơn giản biểu thức để khử dạng vô định.

❶ VÍ DỤ

Ví dụ 1. Tính giới hạn $A=\lim\limits_{x\to 2}\frac{2x^2+3x-14}{x^2-4}$. **Đs:** $A=\frac{11}{4}$.

Lời giải

Ta có $A=\lim\limits_{x\to 2}\frac{2x^2+3x-14}{x^2-4}=\lim\limits_{x\to 2}\frac{2(x-2)(x+\frac{7}{2})}{(x-2)(x+2)}=\lim\limits_{x\to 2}\frac{2x+7}{x+2}=\frac{11}{4}$

! Cần nhớ: $f(x)=ax^2+bx+c=a(x-x_1)(x-x_2)$ với x_1,x_2 là 2 nghiệm của phương trình $f(x)=0$. *Học sinh thường quên nhân thêm* a.

Ví dụ 2. Tính giới hạn $A=\lim\limits_{x\to 2}\frac{2x^3-5x^2-2x-3}{4x^3-13x^2+4x-3}$. **Đs:** $A=\frac{11}{17}$.

Lời giải

$$A=\lim\limits_{x\to 3}\frac{2x^3-5x^2-2x-3}{4x^3-13x^2+4x-3}=\lim\limits_{x\to 3}\frac{(x-3)(2x^2+x+1)}{(x-3)(4x^2-x+1)}=\lim\limits_{x\to 3}\frac{2x^2+x+1}{4x^2-x+1}=\frac{11}{17}$$

Nhận xét: *Bảng chia Hooc – nơ (đầu rơi, nhân tới cộng chéo) như sau:*

Phân tích $2x^3-5x^2-2x-3$ *thành tích số:*

	2	-5	-2	-3
3	2	1	1	0

$\Rightarrow 2x^3-5x^2-2x-3=(x-3)(2x^2+x+1)$

Phân tích $4x^3-13x^2+4x-3$ *thành tích số:*

	4	-13	4	-3
3	4	-1	1	0

$\Rightarrow 4x^3-13x^2+4x-3=(x-3)(4x^2-x+1)$.

Ví dụ 3. Tính giới hạn $A=\lim\limits_{x\to 1}\dfrac{x^{100}-2x+1}{x^{50}-2x+1}$. **Đs:** $A=\dfrac{49}{24}$.

Lời giải

Ta có $A=\lim\limits_{x\to 1}\dfrac{x^{100}-2x+1}{x^{50}-2x+1}=\lim\limits_{x\to 1}\dfrac{(x^{100}-x)-(x-1)}{(x^{50}-x)-(x-1)}=\lim\limits_{x\to 1}\dfrac{x(x^{99}-1)-(x-1)}{x(x^{49}-1)-(x-1)}$

$=\lim\limits_{x\to 1}\dfrac{x(x-1)(x^{98}+x^{97}+x^{96}+....+x+1)-(x-1)}{x(x-1)(x^{48}+x^{47}+x^{46}+....+x+1)-(x-1)}$

$=\lim\limits_{x\to 1}\dfrac{(x-1)(x^{99}+x^{98}+x^{97}+....+x^2+x-1)}{(x-1)(x^{49}+x^{48}+x^{47}+....+x^2+x-1)}$

$=\lim\limits_{x\to 1}\dfrac{(x^{99}+x^{98}+x^{97}+....+x^2+x-1)}{(x^{49}+x^{48}+x^{47}+....+x^2+x-1)}=\dfrac{98}{48}=\dfrac{49}{24}$

!Cần nhớ: *Hằng đẳng thức* $x^n-1=(x-1)(x^{n-1}+x^{n-2}+....+x^2+x+1)$.

Chứng minh: *Xét cấp số nhân* $1,x,x^2,x^3,....,x^{n-1}$ *có* n *số hạng và* $u_1=1, q=x$.

Khi đó

$S_n=1+x+x^2+...+x^{n-1}=u_1\dfrac{q^n-1}{q-1}=1.\dfrac{x^n-1}{x-1}\Leftrightarrow x^n-1=(x-1)(1+x+x^2+...+x^{n-1})$.

❷ BÀI TẬP ÁP DỤNG

Bài 1. Tính các giới hạn sau:

1) $A=\lim\limits_{x\to 2}\dfrac{x^2-3x+2}{x^2-4}$. ĐS: $A=\dfrac{1}{4}$.

2) $A=\lim\limits_{x\to 1}\dfrac{x^2-1}{x^2+3x-4}$. ĐS: $A=\dfrac{2}{5}$.

3) $A=\lim\limits_{x\to 3}\dfrac{x^2-7x+12}{x^2-9}$. ĐS: $A=-\dfrac{1}{6}$.

4) $A=\lim\limits_{x\to 5}\dfrac{x^2-9x+20}{x^2-5x}$. ĐS: $A=\dfrac{1}{5}$.

5) $A=\lim\limits_{x\to 3}\dfrac{3x^2-10x+3}{x^2-5x+6}$. ĐS: $A=8$.

6) $A=\lim\limits_{x\to 1}\dfrac{x^2+2x-3}{2x^2-x-1}$. ĐS: $A=\dfrac{4}{3}$.

7) $A=\lim\limits_{x\to -2}\dfrac{x^4-16}{x^2+6x+8}$. ĐS: $A=-16$.

8) $A=\lim\limits_{x\to 1}\dfrac{x-2\sqrt{x}-3}{x-5\sqrt{x}+4}$.ĐS: $A=-\dfrac{4}{3}$.

9) $A=\lim\limits_{x\to 2}\dfrac{x^3-8}{x^2-3x+2}$. ĐS: $A=12$.

10) $A=\lim\limits_{x\to -2}\dfrac{x^3+8}{x^2+11x+18}$. ĐS: $A=\dfrac{12}{7}$.

Bài 2. Tính các giới hạn sau:

1) $A=\lim\limits_{x\to 1}\frac{2x^3-5x^2+2x+1}{x^2-1}$. ĐS: $A=-1$.

2) $A=\lim\limits_{x\to 1}\frac{x^3-3x+2}{x^4-4x+3}$. ĐS: $A=\frac{1}{2}$.

3) $A=\lim\limits_{x\to -1}\frac{2x^3+5x^2+4x+1}{x^3+x^2-x-1}$. ĐS: $A=\frac{1}{2}$.

4) $A=\lim\limits_{x\to 1}\frac{x^4-x^3-x+1}{x^3-5x^2+7x-3}$. ĐS: $A=-\frac{3}{2}$.

5) $A=\lim\limits_{x\to -\sqrt{3}}\frac{2x^3-3x^2+x+9+7\sqrt{3}}{3-x^2}$. ĐS: $A=\frac{18+19\sqrt{3}}{6}$.

6) $A=\lim\limits_{x\to 3}\frac{x^3-5x^2+3x+9}{x^4-8x^2-9}$. ĐS: $A=0$.

7) $A=\lim\limits_{x\to 1}\frac{1-x^3}{x^4-4x^2+3}$. ĐS: $A=\frac{3}{4}$.

8) $A=\lim\limits_{x\to 2}\left(\frac{1}{x-2}-\frac{12}{x^3-8}\right)$. ĐS: $A=\frac{1}{2}$.

9) $A=\lim\limits_{x\to 2}\left(\frac{1}{x^2-3x-2}+\frac{1}{x^2-5x-6}\right)$. ĐS: $A=-2$.

10) $A=\lim\limits_{x\to 1}\left(\frac{1}{x^2+x-2}-\frac{1}{x^3-1}\right)$. ĐS: $A=\frac{1}{9}$.

Bài 3. Tính các giới hạn sau:

1) $A=\lim\limits_{x\to 1}\frac{x^{20}-2x+1}{x^{30}-2x+1}$. ĐS: $A=\frac{8}{14}$.

2) $A=\lim\limits_{x\to 1}\frac{x^{50}-1}{x^2-3x+2}$. ĐS: $A=-50$.

3) $A=\lim\limits_{x\to 1}\frac{x^n-nx+n-1}{(x-1)^2}$ (Với n là số nguyên). ĐS: $A=\frac{n^2-n}{2}$.

4) $A=\lim\limits_{x\to 1}\frac{x^{n+1}-(n+1)x+n}{(x-1)^2}$. ĐS: $A=\frac{n(n+1)}{2}$.

5) $A=\lim\limits_{x\to 1}\frac{x+x^2+x^3+...+x^n-n}{x+x^2+x^3+...+x^m-m}$ (m,n là số nguyên). ĐS: $A=\frac{n(n+1)}{m(m+1)}$.

6) $A=\lim\limits_{x\to 1}\left(\frac{m}{1-x^m}-\frac{n}{1-x^n}\right)$. ĐS: $A=\frac{m-n}{2}$.

❸ LỜI GIẢI

Bài 1. 1) Ta có $A=\lim\limits_{x\to 2}\frac{x^2-3x+2}{x^2-4}=\lim\limits_{x\to 2}\frac{(x-1)(x-2)}{(x-2)(x+2)}=\lim\limits_{x\to 2}\frac{x-1}{x+2}=\frac{1}{4}$.

2) Ta có $A=\lim\limits_{x\to 1}\frac{x^2-1}{x^2+3x-4}=\lim\limits_{x\to 1}\frac{(x-1)(x+1)}{(x-1)(x+4)}=\lim\limits_{x\to 1}\frac{x+1}{x+4}=\frac{2}{5}$.

3) Ta có $A=\lim\limits_{x\to 3}\frac{x^2-7x+12}{x^2-9}=\lim\limits_{x\to 3}\frac{(x-3)(x-4)}{(x-3)(x+3)}=\lim\limits_{x\to 3}\frac{x-4}{x+3}=-\frac{1}{6}$.

4) Ta có $A=\lim\limits_{x\to 5}\dfrac{x^2-9x+20}{x^2-5x}=\lim\limits_{x\to 5}\dfrac{(x-4)(x-5)}{x(x-5)}=\lim\limits_{x\to 5}\dfrac{x-4}{x}=\dfrac{1}{5}$.

5) Ta có $A=\lim\limits_{x\to 3}\dfrac{3x^2-10x+3}{x^2-5x+6}=\lim\limits_{x\to 3}\dfrac{(3x-1)(x-3)}{(x-2)(x-3)}=\lim\limits_{x\to 3}\dfrac{3x-1}{x-2}=8$.

6) Ta có $A=\lim\limits_{x\to 1}\dfrac{x^2+2x-3}{2x^2-x-1}=\lim\limits_{x\to 1}\dfrac{(x-1)(x+3)}{(x-1)(2x+1)}=\lim\limits_{x\to 1}\dfrac{x+3}{2x+1}=\dfrac{4}{3}$.

7) Ta có $A=\lim\limits_{x\to -2}\dfrac{x^4-16}{x^2+6x+8}=\lim\limits_{x\to -2}\dfrac{(x-2)(x+2)(x^2+4)}{(x+2)(x+4)}=\lim\limits_{x\to -2}\dfrac{(x-2)(x^2+4)}{(x+4)}=-16$.

8) Ta có $A=\lim\limits_{x\to 1}\dfrac{x-2\sqrt{x}-3}{x-5\sqrt{x}+4}=\lim\limits_{x\to 1}\dfrac{(\sqrt{x}-1)(\sqrt{x}+3)}{(\sqrt{x}-1)(\sqrt{x}-4)}=\lim\limits_{x\to 1}\dfrac{(\sqrt{x}+3)}{(\sqrt{x}-4)}=-\dfrac{4}{3}$.

9) Ta có $A=\lim\limits_{x\to 2}\dfrac{x^3-8}{x^2-3x+2}=\lim\limits_{x\to 2}\dfrac{(x-2)(x^2+2x+4)}{(x-2)(x-1)}=\lim\limits_{x\to 2}\dfrac{(x^2+2x+4)}{(x-1)}=12$.

! Cần nhớ: *Hằng đẳng thức* $a^3+b^3=(a+b)(a^2-ab+b^2)$ và $a^3-b^3=(a-b)(a^2+ab+b^2)$.

10) Ta có $A=\lim\limits_{x\to -2}\dfrac{x^3+8}{x^2+11x+18}=\lim\limits_{x\to -2}\dfrac{(x+2)(x^2-2x+4)}{(x+2)(x+9)}=\lim\limits_{x\to -2}\dfrac{(x^2-2x+4)}{(x+9)}=\dfrac{12}{7}$.

Bài 2. 1) $A=\lim\limits_{x\to 1}\dfrac{2x^3-5x^2+2x+1}{x^2-1}=\lim\limits_{x\to 1}\dfrac{(x-1)(2x^2-3x-1)}{(x-1)(x+1)}=\lim\limits_{x\to 1}\dfrac{2x^2-3x-1}{x+1}=-1$.

2) $A=\lim\limits_{x\to 1}\dfrac{x^3-3x+2}{x^4-4x+3}=\lim\limits_{x\to 1}\dfrac{(x-1)^2(x+2)}{(x-1)^2(x^2+2x+3)}=\lim\limits_{x\to 1}\dfrac{x+2}{x^2+2x+3}=\dfrac{1}{2}$.

3) $A=\lim\limits_{x\to -1}\dfrac{2x^3+5x^2+4x+1}{x^3+x^2-x-1}=\lim\limits_{x\to -1}\dfrac{(x+1)^2(2x+1)}{(x+1)^2(x-1)}=\lim\limits_{x\to -1}\dfrac{2x+1}{x-1}=\dfrac{1}{2}$.

4) $A=\lim\limits_{x\to 1}\dfrac{x^4-x^3-x+1}{x^3-5x^2+7x-3}=\lim\limits_{x\to 1}\dfrac{(x-1)^2(x^2+x+1)}{(x-1)^2(x-3)}=\lim\limits_{x\to 1}\dfrac{x^2+x+1}{x-3}=-\dfrac{3}{2}$.

5) Ta có $A=\lim\limits_{x\to -\sqrt{3}}\dfrac{2x^3-3x^2+x+9+7\sqrt{3}}{3-x^2}=\lim\limits_{x\to -\sqrt{3}}\left(-\dfrac{(x+\sqrt{3})(2x^2-(3+2\sqrt{3})x+7+3\sqrt{3})}{(x+\sqrt{3})(\sqrt{3}-x)}\right)$

$=\lim\limits_{x\to -\sqrt{3}}\left(-\dfrac{2x^2-(3+2\sqrt{3})x+7+3\sqrt{3}}{\sqrt{3}-x}\right)=\dfrac{18+19\sqrt{3}}{6}$.

6) Ta có $A=\lim\limits_{x\to 3}\dfrac{x^3-5x^2+3x+9}{x^4-8x^2-9}=\lim\limits_{x\to 3}\dfrac{(x-1)(x-3)^2}{(x-3)(x+3)(x^2+1)}=\lim\limits_{x\to 3}\dfrac{(x-1)(x-3)}{(x+3)(x^2+1)}=0$.

7) Ta có $A=\lim\limits_{x\to 1}\dfrac{1-x^3}{x^4-4x^2+3}=\lim\limits_{x\to 1}\dfrac{(x-1)(-x^2-x-1)}{(x-1)(x^3+x^2-3x-3)}=\lim\limits_{x\to 1}\dfrac{(-x^2-x-1)}{(x^3+x^2-3x-3)}=\dfrac{3}{4}$.

8) Ta có $A=\lim\limits_{x\to 2}\left(\dfrac{1}{x-2}-\dfrac{12}{x^3-8}\right)=\lim\limits_{x\to 2}\dfrac{x^3-12x+16}{(x-2)(x^3-8)}$

$=\lim\limits_{x\to 2}\dfrac{(x+4)(x-2)^2}{(x-2)^2(x^2+2x+4)}=\lim\limits_{x\to 2}\dfrac{x+4}{x^2+2x+4}=\dfrac{1}{2}$.

9) Ta có $A=\lim\limits_{x\to 2}\left(\dfrac{1}{x^2-3x-2}+\dfrac{1}{x^2-5x-6}\right)=\lim\limits_{x\to 2}\dfrac{x^2-5x-6+x^2-3x-2}{(x^2-3x-2)(x^2-5x-6)}$

$=\lim\limits_{x\to 2}\dfrac{2(x-2)^2}{(x-2)^2(x-3)(x-1)}=\lim\limits_{x\to 2}\dfrac{2}{(x-3)(x-1)}=-2$.

10) Ta có $A=\lim\limits_{x\to 1}\left(\dfrac{1}{x^2+x-2}-\dfrac{1}{x^3-1}\right)=\lim\limits_{x\to 1}\dfrac{x^3-1-x^2-x+2}{(x^2+x-2)(x^3-1)}=\lim\limits_{x\to 1}\dfrac{x^3-x^2-x+1}{(x^2+x-2)(x^3-1)}$

$=\lim\limits_{x\to 1}\dfrac{(x-1)^2(x+1)}{(x-1)^2(x+2)(x^2+x+1)}=\lim\limits_{x\to 1}\dfrac{x+1}{(x+2)(x^2+x+1)}=\dfrac{1}{9}$.

Bài 3. 1) Ta có $A=\lim\limits_{x\to 1}\dfrac{x^{20}-2x+1}{x^{30}-2x+1}=\lim\limits_{x\to 1}\dfrac{x^{20}-x-(x-1)}{x^{30}-x-(x-1)}=\lim\limits_{x\to 1}\dfrac{x(x^{19}-1)-(x-1)}{x(x^{29}-1)-(x-1)}$

$=\lim\limits_{x\to 1}\dfrac{x(x-1)(x^{18}+x^{17}+...+x+1)-(x-1)}{x(x-1)(x^{28}+x^{27}+...+x+1)-(x-1)}=\lim\limits_{x\to 1}\dfrac{(x-1)(x^{19}+x^{18}+...+x-1)}{(x-1)(x^{29}+x^{28}+...+x-1)}$

$=\lim\limits_{x\to 1}\dfrac{(x^{19}+x^{18}+...+x-1)}{(x^{29}+x^{28}+...+x-1)}=\dfrac{18}{28}=\dfrac{9}{24}$.

2) Ta có $A=\lim\limits_{x\to 1}\dfrac{x^{50}-1}{x^2-3x+2}=\lim\limits_{x\to 1}\dfrac{(x-1)(x^{49}+x^{48}+...+x+1)}{(x-1)(x-2)}=\lim\limits_{x\to 1}\dfrac{x^{49}+x^{48}+...+x+1}{x-2}=-50$

3) Ta có $A=\lim\limits_{x\to 1}\dfrac{x^n-nx+n-1}{(x-1)^2}=\lim\limits_{x\to 1}\dfrac{(x^n-1)-n(x-1)}{(x-1)^2}$

$=\lim\limits_{x\to 1}\dfrac{(x-1)(x^{n-1}+x^{n-2}+...+x+1)-n(x-1)}{(x-1)^2}$

$=\lim\limits_{x\to 1}\dfrac{(x-1)(x^{n-1}+x^{n-2}+...+x+1-n)}{(x-1)^2}=\lim\limits_{x\to 1}\dfrac{x^{n-1}+x^{n-2}+...+x+1-n}{x-1}$

$=\lim\limits_{x\to 1}\dfrac{x^{n-1}-1+x^{n-2}-1+...+x^2-1+x-1}{x-1}$

$$=\lim_{x\to 1}\frac{(x-1)(x^{n-2}+x^{n-3}+...+x+1)+(x-1)(x^{n-3}+x^{n-4}+...+x+1)+...+(x-1)}{x-1}$$

$$=\lim_{x\to 1}\left[(x^{n-2}+x^{n-3}+...+x+1)+(x^{n-3}+x^{n-4}+...+x+1)+...+1\right]=(n-1)+(n-2)+...+1=\frac{n^2-n}{2}.$$

4) Ta có $A=\lim_{x\to 1}\frac{x^{n+1}-(n+1)x+n}{(x-1)^2}=\lim_{x\to 1}\frac{(x^{n+1}-x)-n(x-1)}{(x-1)^2}=\lim_{x\to 1}\frac{x(x^n-1)-n(x-1)}{(x-1)^2}$

$$=\lim_{x\to 1}\frac{x(x-1)(x^{n-1}+x^{n-2}+...+x+1)-n(x-1)}{(x-1)^2}=\lim_{x\to 1}\frac{(x-1)(x^n+x^{n-1}+...+x-n)}{(x-1)^2}$$

$$=\lim_{x\to 1}\frac{x^n+x^{n-1}+...+x^2+x-n}{x-1}=\lim_{x\to 1}\frac{x^n-1+x^{n-1}-1+...+x^2-1+x-1}{x-1}$$

$$=\lim_{x\to 1}\frac{(x-1)(x^{n-1}+x^{n-2}+...+x+1)+(x-1)(x^{n-2}+x^{n-3}+...+x+1)+...+(x-1)}{x-1}$$

$$=\lim_{x\to 1}\left[(x^{n-1}+x^{n-2}+...+x+1)+(x^{n-2}+x^{n-3}+...+x+1)+...+1\right]$$

$$=n+(n-1)+(n-2)+...+1=\frac{n(n+1)}{2}.$$

5) Ta có $A=\lim_{x\to 1}\frac{x+x^2+x^3+...+x^n-n}{x+x^2+x^3+...+x^m-m}=\lim_{x\to 1}\frac{x^n-1+x^{n-1}-1+...+x^2-1+x-1}{x^m-1+x^{m-1}-1+...+x^2-1+x-1}$

$$=\lim_{x\to 1}\frac{(x-1)(x^{n-1}+x^{n-2}+...+x+1)+(x-1)(x^{n-2}+x^{n-3}+...+x+1)+...+(x-1)}{(x-1)(x^{m-1}+x^{m-2}+...+x+1)+(x-1)(x^{m-2}+x^{m-3}+...+x+1)+...+(x-1)}$$

$$=\lim_{x\to 1}\frac{(x^{n-1}+x^{n-2}+...+x+1)+(x^{n-2}+x^{n-3}+...+x+1)+...+1}{(x^{m-1}+x^{m-2}+...+x+1)+(x^{m-2}+x^{m-3}+...+x+1)+...+1}$$

$$=\lim_{x\to 1}\frac{n+(n-1)+(n-2)+...+1}{m+(m-1)+(m-2)+...+1}=\frac{n(n+1)}{m(m+1)}.$$

6) Ta có $A=\lim_{x\to 1}\left(\frac{m}{1-x^m}-\frac{n}{1-x^n}\right)=\lim_{x\to 1}\left[\left(\frac{m}{1-x^m}-\frac{1}{1-x}\right)-\left(\frac{n}{1-x^n}-\frac{1}{1-x}\right)\right]$

$$=\lim_{x\to 1}\left(\frac{m}{1-x^m}-\frac{1}{1-x}\right)-\lim_{x\to 1}\left(\frac{n}{1-x^n}-\frac{1}{1-x}\right)$$

Và $\lim_{x\to 1}\left(\frac{m}{1-x^m}-\frac{1}{1-x}\right)=\lim_{x\to 1}\frac{m-(1+x+x^2+...+x^{m-1})}{1-x^m}=\lim_{x\to 1}\frac{(1-x)+(1-x^2)+...+(1-x^{m-1})}{1-x^m}$

$$=\lim_{x\to 1}\frac{(1-x)\left[1+(1+x)+....+(1+x+x^2+...+x^{m-2})\right]}{(1-x)(1+x+x^2+...+x^{m-1})}$$

$$=\lim_{x\to 1}\frac{1+(1+x)+....+(1+x+x^2+...+x^{m-2})}{1+x+x^2+...+x^{m-1}}=\frac{1+2+3+...+m-1}{m}=\frac{m-1}{2}$$

Tương tự ta có $\lim_{x\to 1}\left(\frac{n}{1-x^n}-\frac{1}{1-x}\right)=\frac{n-1}{2}$

Vậy $\lim_{x\to 1}\left(\frac{m}{1-x^m}-\frac{n}{1-x^n}\right)=\frac{m-1}{2}-\frac{n-1}{2}=\frac{m-n}{2}.$

◆Dạng 2. Tính giới hạn vô định dạng $\frac{0}{0}$, trong đó tử thức và mẫu thức có chứa căn thức.

Phương pháp giải:

Nhân lượng liên hợp để khử dạng vô định.

❶ VÍ DỤ

Ví dụ 1. Tính giới hạn $B=\lim\limits_{x\to 6}\frac{3-\sqrt{x+3}}{x-6}$. **Đs:** $B=-\frac{1}{6}$.

Lời giải

Ta có: $B=\lim\limits_{x\to 6}\frac{3-\sqrt{x+3}}{x-6}=\lim\limits_{x\to 6}\frac{\left(3-\sqrt{x+3}\right)\left(3+\sqrt{x+3}\right)}{(x-6)\left(3+\sqrt{x+3}\right)}$

$$=\lim\limits_{x\to 6}\frac{9-(x+3)}{(x-6)\left(3+\sqrt{x+3}\right)}=\lim\limits_{x\to 6}\frac{6-x}{(x-6)\left(3+\sqrt{x+3}\right)}=\lim\limits_{x\to 6}\frac{-1}{3+\sqrt{x+3}}=\frac{-1}{3+\sqrt{6+3}}=-\frac{1}{6}$$

Ví dụ 2. Tính giới hạn $E=\lim\limits_{x\to 2}\frac{\sqrt[3]{3x+2}-\sqrt{5x-6}}{x-2}$. **Đs:** $E=-1$.

Lời giải

Ta có $E=\lim\limits_{x\to 2}\frac{\left(\sqrt[3]{3x+2}-2\right)+\left(2-\sqrt{5x-6}\right)}{x-2}=\underbrace{\lim\limits_{x\to 2}\frac{\sqrt[3]{3x+2}-2}{x-2}}_{A}+\underbrace{\lim\limits_{x\to 2}\frac{2-\sqrt{5x-6}}{x-2}}_{B}$

$$A=\lim\limits_{x\to 2}\frac{\sqrt[3]{3x+2}-2}{x-2}=\lim\limits_{x\to 2}\frac{(3x+2)-8}{(x-2)\left(\sqrt[3]{(3x+2)^2}+2.\sqrt[3]{3x+2}+4\right)}$$

$$=\lim\limits_{x\to 2}\frac{3(x-2)}{(x-2)\left(\sqrt[3]{(3x+2)^2}+2.\sqrt[3]{3x+2}+4\right)}==\lim\limits_{x\to 2}\frac{3}{\sqrt[3]{(3x+2)^2}+2.\sqrt[3]{3x+2}+4}=\frac{1}{4}$$

$$B=\lim\limits_{x\to 2}\frac{2-\sqrt{5x-6}}{x-2}=\lim\limits_{x\to 2}\frac{4-(5x-6)}{(x-2)\left(2+\sqrt{5x-6}\right)}=\lim\limits_{x\to 2}\frac{5(2-x)}{(x-2)\left(2+\sqrt{5x-6}\right)}$$

$$=\lim\limits_{x\to 2}\frac{-5}{(x-2)\left(2+\sqrt{5x-6}\right)}=-\frac{5}{4}$$

Suy ra $E=A+B=\frac{1}{4}-\frac{5}{4}=-1$.

Ví dụ 3. Tính giới hạn $L=\lim\limits_{x\to -1}\frac{\sqrt[3]{5x-3}+2}{x+1}$. **Đs:** $L=\frac{5}{12}$.

Lời giải

Ta có: $L=\lim\limits_{x\to -1}\dfrac{\sqrt[3]{5x-3}+2}{x+1}=\lim\limits_{x\to -1}\dfrac{(5x-3)+8}{(x+1)\left(\sqrt[3]{(5x-3)^2}-2.\sqrt[3]{5x-3}+4\right)}$

$=\lim\limits_{x\to -1}\dfrac{5(x+1)}{(x+1)\left(\sqrt[3]{(5x-3)^2}-2.\sqrt[3]{5x-3}+4\right)}\lim\limits_{x\to -1}\dfrac{5}{\sqrt[3]{(5x-3)^2}-2.\sqrt[3]{5x-3}+4}=\dfrac{5}{12}.$

Ví dụ 4. Tính giới hạn $E=\lim\limits_{x\to 2}\dfrac{\sqrt[3]{3x+2}-\sqrt{3x-2}}{x-2}$. **Đs:** $E=\dfrac{-1}{2}$.

Lời giải

Ta có $E=\lim\limits_{x\to 2}\dfrac{\left(\sqrt[3]{3x+2}-2\right)-\left(\sqrt{3x-2}-2\right)}{x-2}=\lim\limits_{x\to 2}\dfrac{\sqrt[3]{3x+2}-2}{x-2}-\lim\limits_{x\to 2}\dfrac{\sqrt{3x-2}-2}{x-2}$

$=\lim\limits_{x\to 2}\dfrac{(3x+2)-8}{(x-2)\left(\sqrt[3]{(3x+2)^2}+2.\sqrt[3]{3x+2}+4\right)}-\lim\limits_{x\to 2}\dfrac{(3x-2)-4}{(x-2)\left(\sqrt{3x-2}+2\right)}$

$=\lim\limits_{x\to 2}\dfrac{3(x-2)}{(x-2)\left(\sqrt[3]{(3x+2)^2}+2.\sqrt[3]{3x+2}+4\right)}-\lim\limits_{x\to 2}\dfrac{3(x-2)}{(x-2)\left(\sqrt{3x-2}+2\right)}$

$=\lim\limits_{x\to 2}\dfrac{3}{\sqrt[3]{(3x+2)^2}+2.\sqrt[3]{3x+2}+4}-\lim\limits_{x\to 2}\dfrac{3}{\sqrt{3x-2}+2}=\dfrac{1}{4}-\dfrac{3}{4}=-\dfrac{1}{2}.$

Ví dụ 5. Tính giới hạn $F=\lim\limits_{x\to 0}\dfrac{\sqrt{1+2x}.\sqrt[3]{1+4x}-1}{x}$. **Đs:** $F=\dfrac{7}{3}$.

Lời giải

$F=\lim\limits_{x\to 0}\dfrac{\sqrt{1+2x}.\sqrt[3]{1+4x}-1}{x}=\lim\limits_{x\to 0}\dfrac{\sqrt{1+2x}.\left(\sqrt[3]{1+4x}-1\right)+\left(\sqrt{1+2x}-1\right)}{x}$

$=\lim\limits_{x\to 0}\dfrac{\sqrt{1+2x}.\left(\sqrt[3]{1+4x}-1\right)}{x}+\lim\limits_{x\to 0}\dfrac{\sqrt{1+2x}-1}{x}$

$=\lim\limits_{x\to 0}\dfrac{\sqrt{1+2x}.\left((1+4x)-1\right)}{x\left(\sqrt[3]{(1+4x)^2}+\sqrt[3]{1+4x}+1\right)}+\lim\limits_{x\to 0}\dfrac{(1+2x)-1}{x\left(\sqrt{1+2x}-1\right)}$

$=\lim\limits_{x\to 0}\dfrac{4.\sqrt{1+2x}}{\sqrt[3]{(1+4x)^2}+\sqrt[3]{1+4x}+1}+\lim\limits_{x\to 0}\dfrac{2}{\sqrt{1+2x}-1}=\dfrac{4}{3}+1=\dfrac{7}{3}.$

❷ BÀI TẬP ÁP DỤNG

Bài 1. Tính các giới hạn sau:

1) $B=\lim\limits_{x\to 8}\dfrac{x-8}{3-\sqrt{x+1}}$. **Đs:** $B=-6$

2) $B=\lim\limits_{x\to -1}\dfrac{\sqrt{4+x+x^2}-2}{x+1}$. **Đs:** $B=-\dfrac{1}{4}$

3) $B=\lim\limits_{x\to 3}\dfrac{\sqrt{2x^2-3x}-x}{2x-6}$. **Đs:** $B=\dfrac{1}{4}$

4) $B=\lim\limits_{x\to 2}\dfrac{\sqrt{x+2}-2}{x^2-4}$. **Đs:** $B=\dfrac{1}{16}$

5) $B=\lim\limits_{x\to 2}\dfrac{2-\sqrt{3x-2}}{x^2-4}$. **Đs:** $B=-\dfrac{3}{16}$

6) $B=\lim\limits_{x\to 9}\dfrac{\sqrt{x}-3}{9x-x^2}$. **Đs:** $B=-\dfrac{1}{54}$

7) $B=\lim\limits_{x\to 2}\dfrac{\sqrt{x+2}-2}{2x^2+x-10}$. **Đs:** $B=\dfrac{1}{36}$

8) $B=\lim\limits_{x\to -1}\dfrac{\sqrt{7-2x}+x-2}{x^2-1}$. **Đs:** $B=-\dfrac{1}{3}$

9) $B=\lim\limits_{x\to -1}\dfrac{2x+5-\sqrt{2x^2+x+8}}{x^2+3x+2}$. **Đs:** $B=\dfrac{5}{2}$

Bài 2. Tính các giới hạn sau:

1) $B=\lim\limits_{x\to 1}\dfrac{\sqrt{3x+1}-\sqrt{x+3}}{\sqrt{x+8}-3}$. **Đs:** $B=3$

2) $B=\lim\limits_{x\to 1}\dfrac{\sqrt{x+3}-2}{\sqrt{4x+5}-\sqrt{3x+6}}$. **Đs:** $B=\dfrac{3}{2}$

3) $B=\lim\limits_{x\to 2}\dfrac{\sqrt{x+2}-\sqrt{2x}}{\sqrt{x-1}-\sqrt{3-x}}$. **Đs:** $B=-\dfrac{1}{4}$

4) $B=\lim\limits_{x\to 3}\dfrac{\sqrt{x+1}-\sqrt{3x-5}}{\sqrt{2x+3}-\sqrt{x+6}}$. **Đs:** $B=-3$

5) $B=\lim\limits_{x\to -1}\dfrac{\sqrt{x^2+x+2}-\sqrt{1-x}}{x^4+x}$. **Đs:** $B=0$

6) $B=\lim\limits_{x\to 1}\dfrac{\sqrt[4]{4x-3}-1}{x-1}$. **Đs:** $B=1$

7) $B=\lim\limits_{x\to 2}\dfrac{\sqrt{2x^2+1}-\sqrt{2x+5}}{\sqrt{x^2+1}-\sqrt{x+3}}$. **Đs:** $B=\dfrac{2\sqrt{5}}{3}$

Bài 3. Tính các giới hạn sau:

1) $L=\lim\limits_{x\to 0}\dfrac{\sqrt{x+9}+\sqrt{x+16}-7}{x}$. **Đs:** $B=\dfrac{7}{24}$

2) $L=\lim\limits_{x\to 1}\dfrac{\sqrt{2x+2}+\sqrt{5x+4}-5}{x-1}$. **Đs:** $B=\dfrac{4}{3}$

3) $L=\lim\limits_{x\to 3}\dfrac{2\sqrt{x+6}+\sqrt{2x-2}-8}{x-3}$. **Đs:** $L=\dfrac{5}{6}$

4) $L=\lim\limits_{x\to 2}\dfrac{2x\sqrt{x-1}+x^2-8}{x-2}$. **Đs:** $L=8$

5) $L=\lim\limits_{x\to 6}\dfrac{5x-4\ \sqrt{2x-3}+x-84}{x-6}$. **Đs:** $L=\dfrac{74}{3}$

6) $L=\lim\limits_{x\to 0}\dfrac{\sqrt{1+4x}\sqrt{1+6x}-1}{x}$. **Đs:** $L=5$

7) $L=\lim\limits_{x\to 0}\dfrac{\sqrt{4x-3}+\sqrt{2x-1}-3x+1}{x^2-2x+1}$. **Đs:** $L=-\dfrac{5}{2}$

8) $L=\lim\limits_{x\to 1}\dfrac{-3x-7+4\sqrt{x+3}+2\sqrt{2x-1}}{x^2-2x+1}$. **Đs:** $L=-\dfrac{17}{16}$

9) $L=\lim\limits_{x\to 0}\dfrac{\sqrt{4x+4}+\sqrt{9-6x}-5}{x^2}$. **Đs:** $L=-\dfrac{5}{12}$

10) $L=\lim\limits_{x\to 1}\dfrac{\sqrt{6x+3}+2x^2-5x}{(x-1)^2}$. **Đs:** $L=\dfrac{11}{6}$

Bài 4. Tính các giới hạn sau:

1) $L=\lim\limits_{x\to 2}\dfrac{\sqrt[3]{4x}-2}{x-2}$. **Đs:** $L=\dfrac{1}{3}$

2) $L=\lim\limits_{x\to 0}\dfrac{1-\sqrt[3]{1-x}}{x}$. **Đs:** $L=\dfrac{1}{3}$

3) $L=\lim\limits_{x\to 3}\dfrac{\sqrt[3]{x^2-1}-2}{x-3}$. **Đs:** $L=\dfrac{1}{2}$

4) $L=\lim\limits_{x\to 1}\dfrac{\sqrt[3]{x+7}-2}{\sqrt{x}-1}$. **Đs:** $L=\dfrac{1}{6}$

5) $L=\lim\limits_{x\to 8}\dfrac{\sqrt[3]{x}-2}{\sqrt{2x+9}-5}$. **Đs:** $L=\dfrac{5}{12}$

6) $L=\lim\limits_{x\to 1}\dfrac{\sqrt[3]{x}-1}{\sqrt[3]{x-2}+1}$. **Đs:** $L=1$

7) $L=\lim\limits_{x\to -1}\dfrac{\sqrt[3]{10+2x^3}+x-1}{x^2+3x+2}$. **Đs:** $L=\dfrac{3}{2}$

8) $L=\lim\limits_{x\to 2}\dfrac{\sqrt[3]{8x+11}-\sqrt{x+7}}{x^2-3x+2}$. **Đs:** $L=\dfrac{7}{54}$

9) $L=\lim\limits_{x\to 1}\dfrac{\sqrt[3]{x^3+7}-\sqrt{x^2+3}}{x-1}$. **Đs:** $L=-\dfrac{1}{4}$

10) $L=\lim\limits_{x\to 0}\dfrac{2\sqrt{1-x}-\sqrt[3]{8-x}}{x}$. **Đs:** $L=-\dfrac{11}{12}$

11) $L=\lim\limits_{x\to 2}\dfrac{\sqrt[3]{2x^2+4x+11}-\sqrt{x+7}}{x^2-4}$. **Đs:** $L=\dfrac{5}{72}$

12) $L=\lim\limits_{x\to 0}\dfrac{\sqrt{4+x}.\sqrt[3]{8+3x}-4}{x^2+x}$. **Đs:** $L=1$

Bài 5. Tính các giới hạn sau:

1) $F=\lim\limits_{x\to 0}\dfrac{\sqrt[n]{1+ax}-1}{x}$. **Đs:** $\dfrac{a}{n}$

2) $F=\lim\limits_{x\to 0}\dfrac{\sqrt[n]{1+ax}-\sqrt[m]{1+bx}}{x}$. **Đs:** $\dfrac{a}{n}-\dfrac{b}{m}$

3) $F=\lim\limits_{x\to 0}\dfrac{\sqrt[n]{1+ax}-1}{\sqrt[m]{1+bx}-1}(ab\neq 0)$. **Đs:** $\dfrac{am}{bn}$

4) $F=\lim\limits_{x\to 0}\dfrac{\sqrt[n]{1+ax}-\sqrt[m]{1+bx}}{\sqrt{1+x}-1}$. **Đs:** $2\left(\dfrac{a}{n}-\dfrac{b}{m}\right)$

❸ LỜI GIẢI

Bài 1. 1) $B=\lim\limits_{x\to 8}\dfrac{x-8}{3-\sqrt{x+1}}=\lim\limits_{x\to 8}\dfrac{(x-8)\left(3+\sqrt{x+1}\right)}{\left(3-\sqrt{x+1}\right)\left(3+\sqrt{x+1}\right)}=\lim\limits_{x\to 8}\dfrac{(x-8)\left(3+\sqrt{x+1}\right)}{9-(x+1)}$

$=\lim\limits_{x\to 8}\dfrac{(x-8)\left(3+\sqrt{x+1}\right)}{8-x}=\lim\limits_{x\to 8}\left[-\left(3+\sqrt{x+1}\right)\right]=-6$.

2) $B=\lim\limits_{x\to -1}\dfrac{\sqrt{4+x+x^2}-2}{x+1}=\lim\limits_{x\to -1}\dfrac{\left(\sqrt{4+x+x^2}-2\right)\left(\sqrt{4+x+x^2}+2\right)}{(x+1)\left(\sqrt{4+x+x^2}+2\right)}$

$=\lim\limits_{x\to -1}\dfrac{4+x+x^2-4}{(x+1)\left(\sqrt{4+x+x^2}+2\right)}=\lim\limits_{x\to -1}\dfrac{x(x+1)}{(x+1)\left(\sqrt{4+x+x^2}+2\right)}=\lim\limits_{x\to -1}\dfrac{x}{\sqrt{4+x+x^2}+2}=-\dfrac{1}{4}$.

3) $B=\lim\limits_{x\to 3}\dfrac{\sqrt{2x^2-3x}-x}{2x-6}=\lim\limits_{x\to 3}\dfrac{\left(\sqrt{2x^2-3x}-x\right)\left(\sqrt{2x^2-3x}+x\right)}{(2x-6)\left(\sqrt{2x^2-3x}+x\right)}$

$=\lim\limits_{x\to 3}\dfrac{x(x-3)}{2(x-3)\left(\sqrt{2x^2-3x}+x\right)}=\lim\limits_{x\to 3}\dfrac{x}{2\left(\sqrt{2x^2-3x}+x\right)}=\dfrac{1}{4}$.

4) $B=\lim\limits_{x\to 2}\dfrac{\sqrt{x+2}-2}{x^2-4}=\lim\limits_{x\to 2}\dfrac{\left(\sqrt{x+2}-2\right)\left(\sqrt{x+2}+2\right)}{\left(x^2-4\right)\left(\sqrt{x+2}+2\right)}$

$=\lim\limits_{x\to 2}\dfrac{x-2}{(x-2)(x+2)\left(\sqrt{x+2}+2\right)}=\lim\limits_{x\to 2}\dfrac{1}{(x+2)\left(\sqrt{x+2}+2\right)}=\dfrac{1}{16}$.

5) $B=\lim\limits_{x\to 2}\dfrac{2-\sqrt{3x-2}}{x^2-4}=\lim\limits_{x\to 2}\dfrac{\left(2-\sqrt{3x-2}\right)\left(2+\sqrt{3x-2}\right)}{\left(x^2-4\right)\left(2+\sqrt{3x-2}\right)}=\lim\limits_{x\to 2}\dfrac{4-(3x-2)}{\left(x^2-4\right)\left(2+\sqrt{3x-2}\right)}$

$=\lim\limits_{x\to 2}\dfrac{3(2-x)}{(x-2)(x+2)\left(2+\sqrt{3x-2}\right)}=\lim\limits_{x\to 2}\dfrac{-3}{(x+2)\left(2+\sqrt{3x-2}\right)}=-\dfrac{3}{16}$.

6) $B=\lim\limits_{x\to 9}\dfrac{\sqrt{x}-3}{9x-x^2}=\lim\limits_{x\to 9}\dfrac{\left(\sqrt{x}-3\right)\left(\sqrt{x}+3\right)}{\left(9x-x^2\right)\left(\sqrt{x}+3\right)}=\lim\limits_{x\to 9}\dfrac{x-9}{x(9-x)\left(\sqrt{x}+3\right)}=\lim\limits_{x\to 9}\dfrac{-1}{x\left(\sqrt{x}+3\right)}=-\dfrac{1}{54}$.

7) $B=\lim\limits_{x\to2}\dfrac{\sqrt{x+2}-2}{2x^2+x-10}=\lim\limits_{x\to2}\dfrac{x-2}{(x-2)(2x+5)\left(\sqrt{x+2}+2\right)}=\lim\limits_{x\to2}\dfrac{1}{(2x+5)\left(\sqrt{x+2}+2\right)}=\dfrac{1}{36}.$

8) $B=\lim\limits_{x\to-1}\dfrac{\sqrt{7-2x}+x-2}{x^2-1}=\lim\limits_{x\to-1}\dfrac{7-2x-(x-2)^2}{(x^2-1)\left(\sqrt{7-2x}+2-x\right)}=\lim\limits_{x\to-1}\dfrac{-x^2+2x+3}{(x^2-1)\left(\sqrt{7-2x}+2-x\right)}$

$=\lim\limits_{x\to-1}\dfrac{(x+1)(3-x)}{(x+1)(x-1)\left(\sqrt{7-2x}+2-x\right)}=\lim\limits_{x\to-1}\dfrac{3-x}{(x-1)\left(\sqrt{7-2x}+2-x\right)}=-\dfrac{1}{3}.$

9) $B=\lim\limits_{x\to-1}\dfrac{2x+5-\sqrt{2x^2+x+8}}{x^2+3x+2}=\lim\limits_{x\to-1}\dfrac{(2x+5)^2-(2x^2+x+8)}{(x^2+3x+2)\left(2x+5+\sqrt{2x^2+x+8}\right)}$

$=\lim\limits_{x\to-1}\dfrac{2x^2+19x+17}{(x^2+3x+2)\left(2x+5+\sqrt{2x^2+x+8}\right)}=\lim\limits_{x\to-1}\dfrac{(x+1)(2x+17)}{(x+1)(x+2)\left(2x+5+\sqrt{2x^2+x+8}\right)}$

$=\lim\limits_{x\to-1}\dfrac{2x+17}{(x+2)\left(2x+5+\sqrt{2x^2+x+8}\right)}=\dfrac{5}{2}.$

Bài 2. 1) $B=\lim\limits_{x\to1}\dfrac{\sqrt{3x+1}-\sqrt{x+3}}{\sqrt{x+8}-3}=\lim\limits_{x\to1}\dfrac{2(x-1)\left(\sqrt{x+8}+3\right)}{(x-1)\left(\sqrt{3x+1}+\sqrt{x+3}\right)}=\lim\limits_{x\to1}\dfrac{2\left(\sqrt{x+8}+3\right)}{\sqrt{3x+1}+\sqrt{x+3}}=3$

2) $B=\lim\limits_{x\to1}\dfrac{\sqrt{x+3}-2}{\sqrt{4x+5}-\sqrt{3x+6}}=\lim\limits_{x\to1}\dfrac{(x-1)\left(\sqrt{4x+5}+\sqrt{3x+6}\right)}{(x-1)\left(\sqrt{x+3}+2\right)}$

$=\lim\limits_{x\to1}\dfrac{\sqrt{4x+5}+\sqrt{3x+6}}{\sqrt{x+3}+2}=\dfrac{3}{2}.$

3) $B=\lim\limits_{x\to2}\dfrac{\sqrt{x+2}-\sqrt{2x}}{\sqrt{x-1}-\sqrt{3-x}}=\lim\limits_{x\to2}\dfrac{(2-x)\left(\sqrt{x-1}+\sqrt{3-x}\right)}{2(x-2)\left(\sqrt{x+2}+\sqrt{2x}\right)}=\lim\limits_{x\to2}\dfrac{-\sqrt{x-1}-\sqrt{3-x}}{2\left(\sqrt{x+2}+\sqrt{2x}\right)}=-\dfrac{1}{4}.$

4) $B=\lim\limits_{x\to3}\dfrac{\sqrt{x+1}-\sqrt{3x-5}}{\sqrt{2x+3}-\sqrt{x+6}}=\lim\limits_{x\to3}\dfrac{2(3-x)\left(\sqrt{2x+3}+\sqrt{x+6}\right)}{(x-3)\left(\sqrt{x+1}+\sqrt{3x-5}\right)}$

$=\lim\limits_{x\to3}\dfrac{-2\left(\sqrt{2x+3}+\sqrt{x+6}\right)}{\sqrt{x+1}+\sqrt{3x-5}}=-3.$

5) $B=\lim\limits_{x\to-1}\dfrac{\sqrt{x^2+x+2}-\sqrt{1-x}}{x^4+x}=\lim\limits_{x\to-1}\dfrac{(x^2+x+2)-(1-x)}{x(x+1)(x^2-x+2)\left(\sqrt{x^2+x+2}+\sqrt{1-x}\right)}$

$=\lim\limits_{x\to-1}\dfrac{(x+1)^2}{x(x+1)(x^2-x+2)\left(\sqrt{x^2+x+2}+\sqrt{1-x}\right)}=\lim\limits_{x\to-1}\dfrac{x+1}{x(x^2-x+2)\left(\sqrt{x^2+x+2}+\sqrt{1-x}\right)}$

$=0.$

6) $B=\lim\limits_{x\to1}\dfrac{\sqrt[4]{4x-3}-1}{x-1}=\lim\limits_{x\to1}\dfrac{4\left(x-1\right)}{\left(x-1\right)\left(\sqrt[4]{\left(4x-3\right)^3}+\sqrt[4]{\left(4x-3\right)^2}+\sqrt[4]{4x-3}+1\right)}$

$=\lim\limits_{x\to1}\dfrac{4}{\sqrt[4]{\left(4x-3\right)^3}+\sqrt[4]{\left(4x-3\right)^2}+\sqrt[4]{4x-3}+1}=1.$

7) $B=\lim\limits_{x\to2}\dfrac{\sqrt{2x^2+1}-\sqrt{2x+5}}{\sqrt{x^2+1}-\sqrt{x+3}}=\lim\limits_{x\to2}\dfrac{\left(2x^2-2x-4\right)\left(\sqrt{x^2+1}+\sqrt{x+3}\right)}{\left(x^2-x-2\right)\left(\sqrt{2x^2+1}+\sqrt{2x+5}\right)}$

$=\lim\limits_{x\to2}\dfrac{2\left(\sqrt{x^2+1}+\sqrt{x+3}\right)}{\sqrt{2x^2+1}+\sqrt{2x+5}}=\dfrac{2\sqrt{5}}{3}.$

Bài 3. 1) $L=\lim\limits_{x\to0}\dfrac{\sqrt{x+9}+\sqrt{x+16}-7}{x}=\lim\limits_{x\to0}\dfrac{\left(\sqrt{x+9}-3\right)+\left(\sqrt{x+9}-4\right)}{x}$

$=\lim\limits_{x\to0}\dfrac{\dfrac{x}{\sqrt{x+9}+3}+\dfrac{x}{\sqrt{x+16}+4}}{x}=\lim\limits_{x\to0}\left(\dfrac{1}{\sqrt{x+9}+3}+\dfrac{1}{\sqrt{x+16}+4}\right)=\dfrac{7}{24}.$

2) $L=\lim\limits_{x\to1}\dfrac{\sqrt{2x+2}+\sqrt{5x+4}-5}{x-1}=\lim\limits_{x\to1}\dfrac{\left(\sqrt{2x+2}-2\right)+\left(\sqrt{5x+4}-3\right)}{x}$

$=\lim\limits_{x\to1}\dfrac{\dfrac{2\left(x-1\right)}{\sqrt{2x+2}+2}+\dfrac{5\left(x-1\right)}{\sqrt{5x+4}+3}}{x-1}=\lim\limits_{x\to1}\left(\dfrac{2}{\sqrt{2x+2}+2}+\dfrac{5}{\sqrt{5x+4}+3}\right)=\dfrac{4}{3}.$

3) $L=\lim\limits_{x\to3}\dfrac{2\sqrt{x+6}+\sqrt{2x-2}-8}{x-3}=\lim\limits_{x\to3}\dfrac{2\left(\sqrt{x+6}-6\right)+\left(\sqrt{2x-2}-2\right)}{x-3}$

$=\lim\limits_{x\to3}\dfrac{2\dfrac{x+6-9}{\sqrt{x+6}+3}+\dfrac{2x-2-4}{\sqrt{2x-2}+2}}{x-3}=\lim\limits_{x\to3}\dfrac{2\dfrac{x-3}{\sqrt{x+6}+3}+2\dfrac{x-3}{\sqrt{2x-2}+2}}{x-3}$

$=\lim\limits_{x\to3}\left(\dfrac{2}{\sqrt{x+6}+3}+\dfrac{2}{\sqrt{2x-2}+2}\right)=\dfrac{5}{6}.$

4) $L=\lim\limits_{x\to2}\dfrac{2x\sqrt{x-1}+x^2-8}{x-2}=\lim\limits_{x\to2}\dfrac{\left(2x\sqrt{x-1}-x^2\right)+2\left(x^2-4\right)}{x-2}$

$=\lim\limits_{x\to2}\dfrac{x\dfrac{4x-4-x^2}{2\sqrt{x-1}+x}+2\left(x-2\right)\left(x+2\right)}{x-2}=\lim\limits_{x\to2}\dfrac{-x\dfrac{\left(x-2\right)^2}{2\sqrt{x-1}+x}+2\left(x-2\right)\left(x+2\right)}{x-2}$

$=\lim\limits_{x\to2}\left(-x\dfrac{x-2}{2\sqrt{x-1}+x}+2\left(x+2\right)\right)=8.$

5) $L=\lim\limits_{x\to6}\dfrac{\left(5x-4\right)\sqrt{2x-3}+x-84}{x-6}=\lim\limits_{x\to6}\dfrac{\left[\left(5x-4\right)\sqrt{2x-3}-3\left(5x-4\right)\right]+16x-96}{x-6}$

$$=\lim_{x\to 6}\frac{(5x-4)\left(\sqrt{2x-3}-3\right)+16(x-6)}{x-6}=\lim_{x\to 6}\frac{(5x-4)\dfrac{2(x-6)}{\sqrt{2x-3}+3}+16(x-6)}{x-6}$$

$$=\lim_{x\to 6}\left(\frac{10x-8}{\sqrt{2x-3}+3}+16\right)=\frac{74}{3}.$$

6) $$L=\lim_{x\to 0}\frac{\sqrt{1+4x}\sqrt{1+6x}-1}{x}=\lim_{x\to 0}\frac{\sqrt{24x^2+10x+1}-1}{x}=\lim_{x\to 0}\frac{24x^2+10x+1-1}{x\left(\sqrt{24x^2+10x+1}+1\right)}$$

$$=\lim_{x\to 0}\frac{x(24x+10)}{x\left(\sqrt{24x^2+10x+1}+1\right)}=\lim_{x\to 0}\frac{24x+10}{\sqrt{24x^2+10x+1}+1}=5.$$

7) $$L=\lim_{x\to 1}\frac{\sqrt{4x-3}+\sqrt{2x-1}-3x+1}{x^2-2x+1}=\lim_{x\to 1}\left[\frac{\sqrt{2x-1}-x}{(x-1)^2}+\frac{\sqrt{4x-3}-(2x-1)}{(x-1)^2}\right]$$

$$=\lim_{x\to 1}\left[\frac{2x-1-x^2}{(x-1)^2\left(\sqrt{2x-1}+x\right)}+\frac{4x-3-(2x-1)^2}{(x-1)^2\left(\sqrt{4x-3}+2x-1\right)}\right]$$

$$=\lim_{x\to 1}\left[\frac{-1}{\sqrt{2x-1}+x}+\frac{4}{\sqrt{4x-3}+2x-1}\right]=-\frac{5}{2}.$$

8) $$L=\lim_{x\to 1}\frac{-3x-7+4\sqrt{x+3}+2\sqrt{2x-1}}{x^2-2x+1}=\lim_{x\to 1}\frac{\left(4\sqrt{x+3}-(x+7)\right)+\left(2\sqrt{2x-1}-2x\right)}{(x-1)^2}$$

$$=\lim_{x\to 1}\frac{\dfrac{16x+48-x^2-14x-49}{x+7+4\sqrt{x+3}}+\dfrac{4(2x-1)-4x^2}{2x+2\sqrt{2x-1}}}{(x-1)^2}=\lim_{x\to 1}\frac{\dfrac{-(x-1)^2}{x+7+4\sqrt{x+3}}+\dfrac{-4(x-1)^2}{2x+2\sqrt{2x-1}}}{(x-1)^2}$$

$$=\lim_{x\to 1}\left(\frac{-1}{x+7+4\sqrt{x+3}}-\frac{4}{2x+2\sqrt{2x-1}}\right)=-\frac{17}{16}.$$

9) $$L=\lim_{x\to 0}\frac{\sqrt{4x+4}+\sqrt{9-6x}-5}{x^2}=\lim_{x\to 0}\frac{\left(\sqrt{4x+4}-(x+2)\right)+\left(\sqrt{9-6x}+x-3\right)}{x^2}$$

$$=\lim_{x\to 0}\frac{\dfrac{4x+4-x^2-4x-4}{\sqrt{4x+4}+x+2}+\dfrac{9-6x-x^2+6x-9}{\sqrt{9-6x}-(x-3)}}{x^2}=\lim_{x\to 0}\frac{\dfrac{-x^2}{x+2+\sqrt{4x+3}}-\dfrac{-x^2}{\sqrt{9-6x}-(x-3)}}{x^2}$$

$$=\lim_{x\to 0}\left(\frac{-1}{x+2+\sqrt{4x+4}}-\frac{1}{\sqrt{9-6x}-(x-3)}\right)=-\frac{5}{12}.$$

10) $L=\lim\limits_{x\to1}\dfrac{\sqrt{6x+3}+2x^2-5x}{(x-1)^2}=\lim\limits_{x\to1}\dfrac{2(x^2-2x+1)+\sqrt{6x+3}}{(x-1)^2}$

$=\lim\limits_{x\to1}\dfrac{2(x-1)^2+\dfrac{6x+3-x^2-4x-4}{\sqrt{6x+3}+x+2}}{(x-1)^2}=\lim\limits_{x\to1}\dfrac{2(x-1)^2-\dfrac{(x-1)^2}{\sqrt{6x+3}+x+2}}{(x-1)^2}$

$=\lim\limits_{x\to1}\left(2-\dfrac{1}{\sqrt{6x+3}+x+2}\right)=\dfrac{11}{6}.$

Bài 4. 1) $L=\lim\limits_{x\to2}\dfrac{\sqrt[3]{4x}-2}{x-2}=\lim\limits_{x\to2}\dfrac{4x-8}{(x-2)\left(\sqrt[3]{16x^2}+2\sqrt[3]{4x}+4\right)}=\lim\limits_{x\to2}\dfrac{4}{\sqrt[3]{16x^2}+2\sqrt[3]{4x}+4}=\dfrac{1}{3}.$

2) $L=\lim\limits_{x\to0}\dfrac{1-\sqrt[3]{1-x}}{x}=\lim\limits_{x\to0}\dfrac{1-(1-x)}{x\left(1+\sqrt[3]{1-x}+\sqrt[3]{(1-x)^2}\right)}=\lim\limits_{x\to0}\dfrac{1}{1+\sqrt[3]{1-x}+\sqrt[3]{(1-x)^2}}=\dfrac{1}{3}.$

3) $L=\lim\limits_{x\to3}\dfrac{\sqrt[3]{x^2-1}-2}{x-3}=\lim\limits_{x\to3}\dfrac{x^2-9}{(x-3)\left(\sqrt[3]{(x^2-1)^2}+2\sqrt[3]{x^2-1}+4\right)}$

$=\lim\limits_{x\to3}\dfrac{x+3}{\sqrt[3]{(x^2-1)^2}+2\sqrt[3]{x^2-1}+4}=\dfrac{1}{2}.$

4) $L=\lim\limits_{x\to1}\dfrac{\sqrt[3]{x+7}-2}{\sqrt{x}-1}=\lim\limits_{x\to1}\dfrac{x-1}{\dfrac{x-1}{\sqrt{x}+1}.\left(\sqrt[3]{(x+7)^2}+2\sqrt[3]{x+7}+4\right)}$

$=\lim\limits_{x\to1}\dfrac{\sqrt{x}+1}{\sqrt[3]{(x+7)^2}+2\sqrt[3]{x+7}+4}=\dfrac{1}{6}.$

5) $L=\lim\limits_{x\to8}\dfrac{\sqrt[3]{x}-2}{\sqrt{2x+9}-5}=\lim\limits_{x\to8}\dfrac{\dfrac{x-8}{\sqrt[3]{x^2}+2\sqrt[3]{x}+4}}{\dfrac{2x-16}{\sqrt{2x+9}+5}}=\lim\limits_{x\to8}\dfrac{\sqrt{2x+9}+5}{2\left(\sqrt[3]{x^2}+2\sqrt[3]{x}+4\right)}=\dfrac{5}{12}.$

6) $L=\lim\limits_{x\to1}\dfrac{\sqrt[3]{x}-1}{\sqrt[3]{x-2}+1}=\lim\limits_{x\to1}\dfrac{\dfrac{x-1}{\sqrt[3]{x^2}+\sqrt[3]{x}+1}}{\dfrac{x-1}{\sqrt[3]{(x-2)^2}-\sqrt[3]{x-2}+1}}=\lim\limits_{x\to1}\dfrac{\sqrt[3]{(x-2)^2}-\sqrt[3]{x-2}+1}{\sqrt[3]{x^2}+\sqrt[3]{x}+1}=1.$

7) $L=\lim\limits_{x\to-1}\dfrac{\sqrt[3]{10+2x^3}+x-1}{x^2+3x+2}=\lim\limits_{x\to-1}\dfrac{\left(\sqrt[3]{10+2x^3}-2\right)+(x+1)}{(x+1)(x+2)}$

$$= \lim_{x\to -1} \frac{\dfrac{2x^3+2}{\sqrt[3]{(10+2x^3)^2}+2\sqrt[3]{10+2x^3}+4}+(x+1)}{(x+1)(x+2)}$$

$$= \lim_{x\to -1} \frac{\dfrac{2(x+1)(x^2-x+1)}{\sqrt[3]{(10+2x^3)^2}+2\sqrt[3]{10+2x^3}+4}+(x+1)}{(x+1)(x+2)}$$

$$= \lim_{x\to -1} \frac{\dfrac{2(x^2-x+1)}{\sqrt[3]{(10+2x^3)^2}+2\sqrt[3]{10+2x^3}+4}+1}{x+2} = \frac{3}{2}.$$

8) $L = \lim_{x\to 2} \frac{\sqrt[3]{8x+11}-\sqrt{x+7}}{x^2-3x+2} = \lim_{x\to 2} \frac{\sqrt[3]{8x+11}-3}{x^2-3x+2} - \lim_{x\to 2} \frac{\sqrt{x+7}-3}{x^2-3x+2}$

$$= \lim_{x\to 2} \frac{8x+11-27}{(x-1)(x-2)\left(\sqrt[3]{(8x+11)^2}+3\sqrt[3]{8x+11}+9\right)} - \lim_{x\to 2} \frac{x+7-9}{(x-1)(x-2)\left(\sqrt{x+7}+3\right)}$$

$$= \lim_{x\to 2} \frac{8}{(x-1)\left(\sqrt[3]{(8x+11)^2}+3\sqrt[3]{8x+11}+9\right)} - \lim_{x\to 2} \frac{1}{(x-1)\left(\sqrt{x+7}+3\right)} = \frac{8}{27}-\frac{1}{7} = \frac{7}{54}.$$

9) $L = \lim_{x\to 1} \frac{\sqrt[3]{x^3+7}-\sqrt{x^2+3}}{x-1} = \lim_{x\to 1} \frac{\sqrt[3]{x^3+7}-2}{x-1} - \lim_{x\to 1} \frac{\sqrt{x^2+3}-2}{x-1}$

$$= \lim_{x\to 1} \frac{x^3+7-8}{(x-1)\left(\sqrt[3]{(x^3+7)^2}+2\sqrt[3]{x^3+7}+4\right)} - \lim_{x\to 1} \frac{x^2+3-4}{(x-1)\left(\sqrt{x^2+3}+2\right)}$$

$$= \lim_{x\to 1} \frac{x^3-1}{(x-1)\left(\sqrt[3]{(x^3+7)^2}+2\sqrt[3]{x^3+7}+4\right)} - \lim_{x\to 1} \frac{x^2-1}{(x-1)\left(\sqrt{x^2+3}+2\right)}$$

$$= \lim_{x\to 1} \frac{x^2+x+1}{\sqrt[3]{(x^3+7)^2}+2\sqrt[3]{x^3+7}+4} - \lim_{x\to 1} \frac{x+1}{\sqrt{x^2+3}+2} = \frac{1}{4}-\frac{1}{2} = -\frac{1}{4}.$$

10) $L = \lim_{x\to 0} \frac{2\sqrt{1-x}-\sqrt[3]{8-x}}{x} = \lim_{x\to 0} \frac{2\sqrt{1-x}-2}{x} - \lim_{x\to 0} \frac{\sqrt[3]{8-x}-2}{x}$

$$= \lim_{x\to 0} \frac{4(1-x)-4}{x\left(2\sqrt{1-x}+2\right)} - \lim_{x\to 0} \frac{8-x-8}{x\left(\sqrt[3]{(8-x)^2}+2\sqrt[3]{8-x}+4\right)}$$

$$= \lim_{x\to 0} \frac{-4}{2\sqrt{1-x}+2} - \lim_{x\to 0} \frac{-1}{\sqrt[3]{(8-x)^2}+2\sqrt[3]{8-x}+4} = -1-\frac{-1}{12} = -\frac{11}{12}.$$

11) $L=\lim\limits_{x\to 2}\dfrac{\sqrt[3]{2x^2+4x+11}-\sqrt{x+7}}{x^2-4}=\lim\limits_{x\to 2}\dfrac{\sqrt[3]{2x^2+4x+11}-3}{x^2-4}-\lim\limits_{x\to 2}\dfrac{\sqrt{x+7}-3}{x^2-4}$

$=\lim\limits_{x\to 2}\dfrac{2x^2+4x+11-27}{(x^2-4)\left(\sqrt[3]{(2x^2+4x+11)^2}+3\sqrt[3]{2x^2+4x+11}+9\right)}-\lim\limits_{x\to 2}\dfrac{x+7-9}{(x^2-4)(\sqrt{x+7}-3)}$

$=\lim\limits_{x\to 2}\dfrac{2(x-2)(x+4)}{(x^2-4)\left(\sqrt[3]{(2x^2+4x+11)^2}+3\sqrt[3]{2x^2+4x+11}+9\right)}-\lim\limits_{x\to 2}\dfrac{x-2}{(x^2-4)(\sqrt{x+7}-3)}$

$=\lim\limits_{x\to 2}\dfrac{2(x+4)}{(x+2)\left(\sqrt[3]{(2x^2+4x+11)^2}+3\sqrt[3]{2x^2+4x+11}+9\right)}-\lim\limits_{x\to 2}\dfrac{1}{(x+2)(\sqrt{x+7}-3)}$

$=\dfrac{1}{9}-\dfrac{1}{24}=\dfrac{5}{72}$

12) $L=\lim\limits_{x\to 0}\dfrac{\sqrt{4+x}.\sqrt[3]{8+3x}-4}{x^2+x}=\lim\limits_{x\to 0}\dfrac{\sqrt{4+x}.\left(\sqrt[3]{8+3x}-2\right)+2\sqrt{4+x}-4}{x^2+x}$

$=\lim\limits_{x\to 0}\dfrac{\sqrt{4+x}.\left(\sqrt[3]{8+3x}-2\right)}{x^2+x}+\lim\limits_{x\to 0}\dfrac{2\sqrt{4+x}-4}{x^2+x}$

$=\lim\limits_{x\to 0}\dfrac{\sqrt{4+x}.(8+3x-8)}{x(x+1)\left(\sqrt[3]{(8+3x)^2}+2\sqrt[3]{8+3x}+4\right)}+\lim\limits_{x\to 0}\dfrac{2(4+x-4)}{x(x+1)(\sqrt{4+x}+2)}$

$=\lim\limits_{x\to 0}\dfrac{\sqrt{4+x}.3}{(x+1)\left(\sqrt[3]{(8+3x)^2}+2\sqrt[3]{8+3x}+4\right)}+\lim\limits_{x\to 0}\dfrac{2}{(x+1)(\sqrt{4+x}+2)}$

$=\dfrac{1}{2}+\dfrac{1}{2}=1.$

Bài 5. 1) $F=\lim\limits_{x\to 0}\dfrac{\sqrt[n]{1+ax}-1}{x}=\lim\limits_{x\to 0}\dfrac{1+ax-1}{x\left(\sqrt[n]{(1+ax)^{n-1}}+\sqrt[n]{(1+ax)^{n-2}}+...+\sqrt[n]{1+ax}+1\right)}$

$=\lim\limits_{x\to 0}\dfrac{a}{\sqrt[n]{(1+ax)^{n-1}}+\sqrt[n]{(1+ax)^{n-2}}+...+\sqrt[n]{1+ax}+1}=\dfrac{a}{n}.$

2) $F=\lim\limits_{x\to 0}\dfrac{\sqrt[n]{1+ax}-\sqrt[m]{1+bx}}{x}=\lim\limits_{x\to 0}\dfrac{\left(\sqrt[n]{1+ax}-1\right)-\left(\sqrt[m]{1+bx}-1\right)}{x}$

$=\lim\limits_{x\to 0}\dfrac{\sqrt[n]{1+ax}-1}{x}-\lim\limits_{x\to 0}\dfrac{\sqrt[m]{1+bx}-1}{x}=\dfrac{a}{n}-\dfrac{b}{m}.$

3) $F=\lim\limits_{x\to 0}\dfrac{\sqrt[n]{1+ax}-1}{\sqrt[m]{1+bx}-1}=\lim\limits_{x\to 0}\left[\dfrac{\sqrt[n]{1+ax}-1}{x}.\dfrac{1}{\dfrac{\sqrt[m]{1+bx}-1}{x}}\right]$

Xét $A=\lim\limits_{x\to 0}\dfrac{\sqrt[n]{1+ax}-1}{x}=\dfrac{a}{n}$; $B=\lim\limits_{x\to 0}\dfrac{\sqrt[m]{1+bx}-1}{x}=\dfrac{b}{m}$

$\Rightarrow F=\dfrac{a}{n}.\dfrac{1}{\dfrac{b}{m}}=\dfrac{am}{bn}$.

4) $F=\lim\limits_{x\to 0}\dfrac{\sqrt[n]{1+ax}-\sqrt[m]{1+bx}}{\sqrt{1+x}-1}=\lim\limits_{x\to 0}\dfrac{\sqrt[n]{1+ax}-1\;-\;\sqrt[m]{1+bx}-1}{\sqrt{1+x}-1}$

$=\lim\limits_{x\to 0}\dfrac{\sqrt[n]{1+ax}-1}{\sqrt{1+x}-1}-\lim\limits_{x\to 0}\dfrac{\sqrt[m]{1+bx}-1}{\sqrt{1+x}-1}$

$=\lim\limits_{x\to 0}\left(\dfrac{\sqrt[n]{1+ax}-1}{x}.\dfrac{x}{\sqrt{1+x}-1}\right)-\lim\limits_{x\to 0}\left(\dfrac{\sqrt[m]{1+bx}-1}{x}.\dfrac{x}{\sqrt{1+x}-1}\right)$

Ta có $A=\lim\limits_{x\to 0}\dfrac{\sqrt[n]{1+ax}-1}{x}=\dfrac{a}{n}$

$B=\lim\limits_{x\to 0}\dfrac{\sqrt[m]{1+bx}-1}{x}=\dfrac{b}{m}$ $C=\lim\limits_{x\to 0}\dfrac{x}{\sqrt{1+x}-1}=\lim\limits_{x\to 0}\dfrac{x\;\sqrt{1+x}+1}{1+x-1}=\lim\limits_{x\to 0}\;\sqrt{1+x}+1\;=2$

$\Rightarrow F=\dfrac{a}{n}.2-\dfrac{b}{m}.2=2\left(\dfrac{a}{n}-\dfrac{b}{m}\right)$.

◆Dạng 3. Giới hạn của hàm số khi $x\to\infty$.

Phương pháp giải:

- *Đối với dạng đa thức không căn, ta rút bậc cao và áp dụng công thức khi* $x\to+\infty$

1. $\lim\limits_{x\to+\infty}x^k=+\infty$
2. $\lim\limits_{x\to-\infty}x^k=\begin{cases}+\infty\,khi\,k=2l\\-\infty\,khi\,k=2l+1\end{cases}$
3. $\lim\limits_{x\to+\infty}\dfrac{c}{x^k}=0$ (c hằng số)

- *Đối với dạng phân số không căn, ta làm tương tự như giới hạn dãy số, tức rút bậc cao nhất của tử và mẫu, sau đó áp dụng công thức trên.*
- *Ngoài việc đưa ra khỏi căn bậc chẵn cần có trị tuyệt đối, học sinh cần phân biệt khi nào đưa ra ngoài căn, khi nào liên hợp. Phương pháp suy luận cũng tương tự như giới hạn của dãy số, nhưng cần phân biệt khi* $x\to+\infty$ *hoặc* $x\to-\infty$

❶ VÍ DỤ

Ví dụ 1. Tính giới hạn $A=\lim\limits_{x\to+\infty}\left(-x^3-6x^2+9x+1\right)$. **Đs:** $-\infty$.

Lời giải

$A=\lim\limits_{x\to+\infty}x^3\left(-1-\dfrac{6}{x}+\dfrac{9}{x^2}+\dfrac{1}{x^3}\right)=-\infty$ (vì $\lim\limits_{x\to+\infty}x^3=+\infty$ và $\lim\limits_{x\to+\infty}\left(-1-\dfrac{6}{x}+\dfrac{9}{x^2}+\dfrac{1}{x^3}\right)=-1$).

Ví dụ 2. Tính giới hạn $B=\lim\limits_{x\to+\infty}\dfrac{x^3+3x+1}{2-6x^2-6x^3}$. **Đs:** $\dfrac{1}{6}$.

Lời giải

$$B=\lim\limits_{x\to+\infty}\frac{x^3\left(1+\frac{3}{x^2}+\frac{1}{x^3}\right)}{x^3\left(\frac{2}{x^3}-\frac{6}{x}-6\right)}=\lim\limits_{x\to+\infty}\frac{1+\frac{3}{x^2}+\frac{1}{x^3}}{\frac{2}{x^3}-\frac{6}{x}-6}=\frac{1+0+0}{0-0-6}=-\frac{1}{6}.$$

Ví dụ 2. Tính giới hạn $C=\lim\limits_{x\to-\infty}\left(\sqrt{x^2+x+1}+2x\right)$. **Đs:** $-\infty$.

Lời giải

$$C=\lim\limits_{x\to-\infty}\left[\sqrt{x^2\left(1+\frac{1}{x}+\frac{1}{x^2}\right)}+2x\right]=\lim\limits_{x\to-\infty}\left[|x|\sqrt{\left(1+\frac{1}{x}+\frac{1}{x^2}\right)}+2x\right]$$

$$=\lim\limits_{x\to-\infty}\left[-x\sqrt{\left(1+\frac{1}{x}+\frac{1}{x^2}\right)}+2x\right]=\lim\limits_{x\to-\infty}\left[x\left(2-\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}\right)\right]=-\infty$$

(Vì $\lim\limits_{x\to-\infty}x=-\infty$ *và* $\lim\limits_{x\to-\infty}\left[2-\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}\right]=2-1=1>0$*).*

❷ BÀI TẬP ÁP DỤNG

Bài 1. Tính các giới hạn sau:

1) $A=\lim\limits_{x\to-\infty}\left(x^3-3x^2+2\right)$. **Đs:** $-\infty$.

2) $A=\lim\limits_{x\to-\infty}\left(-x^3+3x^2-1\right)$. **Đs:** $+\infty$.

3) $A=\lim\limits_{x\to+\infty}\left(x^4-2x^2+1\right)$. **Đs:** $+\infty$.

4) $A=\lim\limits_{x\to+\infty}\left(-x^4+2x^2+3\right)$. **Đs:** $-\infty$.

5) $A=\lim\limits_{x\to-\infty}\left(-x^4-x^2+6\right)$. **Đs:** $-\infty$.

Bài 2. Tính các giới hạn sau:

1) $B=\lim\limits_{x\to+\infty}\dfrac{1-8x}{2x-1}$. **Đs:** $B=-4$.

2) $B=\lim\limits_{x\to+\infty}\dfrac{x+2}{x-1}$. **Đs:** $B=1$.

3) $B=\lim\limits_{x\to-\infty}\dfrac{2x^4+7x^3-15}{x^4+1}$. **Đs:** $B=2$.

4) $B=\lim\limits_{x\to+\infty}\dfrac{2x^3+3x-4}{-x^3-x^2+1}$. **Đs:** $B=-2$.

5) $B=\lim\limits_{x\to+\infty}\dfrac{3x^2-x+7}{2x^3-1}$. **Đs:** $B=0$.

6) $B=\lim\limits_{x\to+\infty}\left(\dfrac{x^3}{3x^2-4}-\dfrac{x^2}{3x+2}\right)$. **Đs:** $B=\dfrac{2}{9}$.

7) $B=\lim\limits_{x\to+\infty}\dfrac{(4x+3)^3(2x+1)^4}{(2+2x)^7}$. **Đs:** $B=8$.

8) $B=\lim\limits_{x\to-\infty}\dfrac{(2x-3)^{20}(3x+2)^{30}}{(1+2x)^{50}}$. **Đs:** $B=\left(\dfrac{3}{2}\right)^{30}$.

9) $B=\lim\limits_{x\to-\infty}\dfrac{3x^2-x+3}{x-4}$. **Đs:** $B=-\infty$.

10) $B=\lim\limits_{x\to-\infty}\dfrac{2x^3-2x+3}{5-x}$. **Đs:** $B=-\infty$.

Bài 3. Tính các giới hạn sau:

1) $C=\lim\limits_{x\to+\infty}\left(\sqrt{x^2-3x+2}-x+10\right)$. **Đs:** $\dfrac{17}{2}$.

2) $C=\lim\limits_{x\to+\infty}\dfrac{\sqrt{2x^4+x^2-1}}{1-2x}$. **Đs:** $-\infty$.

3) $C=\lim\limits_{x\to-\infty}\left(\sqrt{4x^2-4x+1}+2x+13\right)$. **Đs:** 14.

4) $C=\lim\limits_{x\to-\infty}\left(\sqrt{x^2+x}+x+5\right)$. **Đs:** $\dfrac{9}{2}$.

5) $C=\lim\limits_{x\to-\infty}\left(\sqrt{2x^2+1}+x\right)$. **Đs:** $+\infty$.

6) $C=\lim\limits_{x\to+\infty}\left(\sqrt{x^2-4x}-x+2021\right)$. **Đs:** 2019.

7) $C=\lim\limits_{x\to-\infty}\left(\sqrt{x^2+x}-\sqrt{x^2+1}\right)$. **Đs:** $-\dfrac{1}{2}$.

8) $C=\lim\limits_{x\to-\infty}\dfrac{2|x|+3}{\sqrt{x^2+x+5}}$. **Đs:** -2.

9) $C=\lim\limits_{x\to+\infty}\left(\sqrt{4x^4+3x^2+1}-2x^2\right)$. **Đs:** $\dfrac{3}{4}$.

10) $C=\lim\limits_{x\to-\infty}\dfrac{\sqrt{x^2+x}+2x}{2x+3}$. **Đs:** $\dfrac{1}{2}$.

11) $C=\lim\limits_{x\to+\infty}\left(\sqrt{x^2+x+1}-x+1\right)$. **Đs:** $\dfrac{3}{2}$.

12) $C=\lim\limits_{x\to-\infty}\dfrac{|x|-\sqrt{x^2+x}}{x+10}$. **Đs:** -2.

13) $C=\lim\limits_{x\to-\infty}\sqrt{4x^2-9x-21}-\sqrt{4x^2-7x+13}$. **Đs:** $\dfrac{1}{2}$

14) $C=\lim\limits_{x\to-\infty}\dfrac{4x^2|x|-3x^2+7x-1}{2x+1\ ^2.\sqrt{x^2+3x}}$. **Đs:** 1.

15) $C=\lim\limits_{x\to-\infty}\sqrt{4x^2-4x+1}+2x+3$. **Đs:** 4

16) $C=\lim\limits_{x\to-\infty}\left[\ x-1\ \sqrt{\dfrac{3-x}{5-3x-x^3}}\right]$. **Đs:** -1

17) $C=\lim\limits_{x\to+\infty}\sqrt{16x^2+3x}-4x+5$. **Đs:** $\dfrac{43}{8}$

18) $C=\lim\limits_{x\to-\infty}\left(x\sqrt{\dfrac{2x^3+x}{x^5-x^2+3}}\right)$. **Đs:** $-\sqrt{2}$

19) $C=\lim\limits_{x\to+\infty}\ x-3-\sqrt{x^2-x+1}$. **Đs:** $-\dfrac{5}{2}$

Bài 4. Tính các giới hạn sau:

1) $\lim\limits_{x\to-\infty}\left[x.\sqrt{\dfrac{2x^3+x}{x^5-x^2+3}}\right]$. **Đs:** $-\sqrt{2}$.

2) $\lim\limits_{x\to-\infty}\dfrac{2|x|+3}{\sqrt{x^2+x+5}}$. **Đs:** 2.

3) $\lim\limits_{x\to+\infty}\dfrac{\sqrt{x^2+x+2}+3x+1}{\sqrt{4x^2+1}+1-x}$. **Đs:** 4.

4) $\lim\limits_{x\to-\infty}\dfrac{\sqrt{2x^4-x^2}-x\sqrt{x^2+3}}{x\ \ 5-2x}$ **Đs:** $-\dfrac{\sqrt{2}+1}{2}$.

5) $\lim\limits_{x\to+\infty}\dfrac{2x-1+3\sqrt{4x^2+x-5}}{1-3x+2\sqrt{9x^2-x+10}}$. **Đs:** $\dfrac{8}{3}$.

6) $\lim\limits_{x\to-\infty}\dfrac{3\sqrt{x^2-1}-\sqrt[3]{1-8x^3}}{6x+9}$. **Đs:** $-\dfrac{1}{6}$.

7) $\lim\limits_{x\to-\infty}\dfrac{2x-1\ \sqrt{x^2-3}}{x-5x^2}$. **Đs:** $\dfrac{2}{5}$.

8) $\lim\limits_{x\to-\infty}\dfrac{x-\sqrt{4x^2+3x-1}}{\sqrt{9x^2-x+3}-5x-3}$. **Đs:** $-\dfrac{1}{4}$

9) $\lim\limits_{x\to+\infty}\dfrac{2x+1}{\sqrt{x^2-x}+\sqrt{x^2+x}}$. **Đs:** 1.

10) $\lim\limits_{x\to-\infty}\dfrac{8x+3}{6x+\sqrt{4x^2+x+3}}$. **Đs:** 2.

11) $\lim\limits_{x\to+\infty}\dfrac{\sqrt{x^2+1}-7x+2}{\sqrt{x^2+3x+2}+5x+3}$. **Đs:** -1.

12) $\lim\limits_{x\to-\infty}\dfrac{x+2\sqrt{1-2x}}{1-x}$. **Đs:** -1.

Bài 5. Tính các giới hạn sau:

1) $\lim\limits_{x\to-\infty}\left(\sqrt{x^2+x}-x\right)$. **Đs:** $+\infty$.

2) $\lim\limits_{x\to+\infty}\left(\sqrt{x^2-4x}-x\right)$. **Đs:** -2.

3) $\lim\limits_{x\to+\infty}\left(\sqrt{x+2}-\sqrt{x-2}\right)$. **Đs:** 0.

4) $\lim\limits_{x\to-\infty}\left(\sqrt{x^2+x}-\sqrt{x^2+1}\right)$. **Đs:** $-\dfrac{1}{2}$.

5) $\lim\limits_{x\to+\infty}\left(\sqrt{x^2+4x+1}-x-2\right)$. **Đs:** 0.

6) $\lim\limits_{x\to-\infty}\left(\sqrt{x^2-3x+5}+x+1\right)$. **Đs:** $\dfrac{5}{2}$.

7) $\lim\limits_{x\to+\infty}\left(\sqrt[3]{27x^3-x^2}-3x\right)$. **Đs:** $-\dfrac{1}{27}$.

8) $\lim\limits_{x\to+\infty}\left(2x-\sqrt{4x^2+2x-1}\right)$. **Đs:** $-\dfrac{1}{2}$.

9) $\lim\limits_{x\to+\infty}\left(2x-3-\sqrt{4x^2+4x+3}\right)$. **Đs:** -4.

10) $\lim\limits_{x\to-\infty}\left(\sqrt{4x^4+3x^2+1}-2x^2\right)$. **Đs:** $\dfrac{3}{4}$.

11) $\lim\limits_{x\to+\infty}\left(\sqrt{4x^2+3x+1}-2x+4\right)$. **Đs:** $\dfrac{19}{4}$.

12) $\lim\limits_{x\to-\infty}\left(\sqrt{4x^2-4x+1}+2x+3\right)$. **Đs:** 4.

13) $\lim\limits_{x\to+\infty}\dfrac{x+\sqrt[3]{4x^2-x^3}}{2x-\sqrt{4x^2-3x}}$. **Đs:** $\dfrac{16}{9}$.

14) $\lim\limits_{x\to+\infty}\ \sqrt[3]{8x^3+1}-2x+1$. **Đs:** 1.

❸ LỜI GIẢI

Bài 1. 1) $A=\lim\limits_{x\to-\infty}x^3\left(1-\dfrac{3}{x}+\dfrac{2}{x^3}\right)=-\infty$, *(vì* $\lim\limits_{x\to-\infty}x^3=-\infty$ *và* $\lim\limits_{x\to-\infty}\left(1-\dfrac{3}{x}+\dfrac{2}{x^3}\right)=1>0$*).*

2) $A=\lim\limits_{x\to-\infty}x^3\left(-1+\dfrac{3}{x}-\dfrac{1}{x^3}\right)=+\infty$, $\left(\text{vì } \lim\limits_{x\to-\infty}x^3=-\infty \text{ và } \lim\limits_{x\to-\infty}\left(-1+\dfrac{3}{x}-\dfrac{1}{x^3}\right)=-1<0\right)$.

3) $A=\lim\limits_{x\to+\infty}x^4\left(1-\dfrac{2}{x^2}+\dfrac{1}{x^4}\right)=+\infty$, $\left(\text{vì } \lim\limits_{x\to+\infty}x^4=+\infty \text{ và } \lim\limits_{x\to+\infty}\left(1-\dfrac{2}{x^2}+\dfrac{1}{x^4}\right)=1>0\right)$.

4) $A=\lim\limits_{x\to+\infty}\left[x^4\left(-1+\dfrac{2}{x^2}+\dfrac{3}{x^4}\right)\right]=-\infty$, $\left(\text{vì } \lim\limits_{x\to+\infty}x^4=+\infty \text{ và } \lim\limits_{x\to+\infty}\left(-1+\dfrac{2}{x^2}+\dfrac{3}{x^4}\right)=-1<0\right)$.

5) $A=\lim\limits_{x\to-\infty}\left[x^4\left(-1-\dfrac{1}{x^2}+\dfrac{6}{x^4}\right)\right]=-\infty$, $\left(\text{vì } \lim\limits_{x\to-\infty}x^4=+\infty \text{ và } \lim\limits_{x\to-\infty}\left(-1-\dfrac{1}{x^2}+\dfrac{6}{x^4}\right)=-1<0\right)$.

Bài 2. 1) $B=\lim\limits_{x\to+\infty}\dfrac{1-8x}{2x-1}=\lim\limits_{x\to+\infty}\dfrac{x\left(\dfrac{1}{x}-8\right)}{x\left(2-\dfrac{1}{x}\right)}=\lim\limits_{x\to+\infty}\dfrac{\dfrac{1}{x}-8}{2-\dfrac{1}{x}}=\dfrac{0-8}{2-0}=-4$.

2) $B=\lim\limits_{x\to+\infty}\dfrac{x+2}{x-1}=\lim\limits_{x\to+\infty}\dfrac{x\left(1+\dfrac{2}{x}\right)}{x\left(1-\dfrac{1}{x}\right)}=\lim\limits_{x\to+\infty}\dfrac{1+\dfrac{2}{x}}{1-\dfrac{1}{x}}=\dfrac{1+0}{1-0}=1$.

3) $B=\lim\limits_{x\to-\infty}\dfrac{2x^4+7x^3-15}{x^4+1}=\lim\limits_{x\to-\infty}\dfrac{x^4\left(2+\dfrac{7}{x}-\dfrac{15}{x^4}\right)}{x^4\left(1+\dfrac{1}{x^4}\right)}=\lim\limits_{x\to-\infty}\dfrac{2+\dfrac{7}{x}-\dfrac{15}{x^4}}{1+\dfrac{1}{x^4}}=\dfrac{2+0-0}{1+0}=2$.

4) $B=\lim\limits_{x\to+\infty}\dfrac{2x^3+3x-4}{-x^3-x^2+1}=\lim\limits_{x\to+\infty}\dfrac{x^3\left(2+\dfrac{3}{x^2}-\dfrac{4}{x^3}\right)}{x^3\left(-1-\dfrac{1}{x}+\dfrac{1}{x^3}\right)}=\lim\limits_{x\to+\infty}\dfrac{\left(2+\dfrac{3}{x^2}-\dfrac{4}{x^3}\right)}{\left(-1-\dfrac{1}{x}+\dfrac{1}{x^3}\right)}=\dfrac{2+0-0}{-1-0+0}=-2$.

5) $B=\lim\limits_{x\to+\infty}\dfrac{3x^2-x+7}{2x^3-1}=\lim\limits_{x\to+\infty}\dfrac{x^3\left(\dfrac{3}{x}-\dfrac{1}{x^2}+\dfrac{7}{x^3}\right)}{x^3\left(2-\dfrac{1}{x^3}\right)}=\lim\limits_{x\to+\infty}\dfrac{\left(\dfrac{3}{x}-\dfrac{1}{x^2}+\dfrac{7}{x^3}\right)}{\left(2-\dfrac{1}{x^3}\right)}=\dfrac{0-0+0}{2-0}=0$.

6) $B=\lim\limits_{x\to+\infty}\left(\dfrac{x^3}{3x^2-4}-\dfrac{x^2}{3x+2}\right)=\lim\limits_{x\to+\infty}\dfrac{x^2\ \ 2x+4}{3x^2-4\ \ 3x+2}$

$$=\lim_{x\to+\infty}\frac{x^3\left(2+\frac{4}{x}\right)}{x^3\left(3-\frac{4}{x^2}\right)\left(3+\frac{2}{x}\right)}\lim_{x\to+\infty}\frac{\left(2+\frac{4}{x}\right)}{\left(3-\frac{4}{x^2}\right)\left(3+\frac{2}{x}\right)}=\frac{2+0}{(3-0)(3+0)}=\frac{2}{9}$$

7) $$B=\lim_{x\to+\infty}\frac{(4x+3)^3(2x+1)^4}{(2+2x)^7}=\lim_{x\to+\infty}\frac{\left(4+\frac{3}{x}\right)^3\left(2+\frac{1}{x}\right)^4}{\left(2+\frac{3}{x}\right)^7}=\frac{(4+0)^3(2+0)^4}{(2+0)^7}=8.$$

8) $$B=\lim_{x\to-\infty}\frac{(2x-3)^{20}(3x+2)^{30}}{(1+2x)^{50}}=\lim_{x\to-\infty}\frac{\left(2-\frac{3}{x}\right)^{20}\left(3+\frac{2}{x}\right)^{30}}{\left(2+\frac{1}{x}\right)^{50}}=\frac{(2-0)^{20}(3+0)^{30}}{(2+0)^{50}}=\left(\frac{3}{2}\right)^{30}.$$

9) $$B=\lim_{x\to-\infty}\frac{3x^2-x+3}{x-4}=\lim_{x\to-\infty}\frac{x^2\left(3-\frac{1}{x}+\frac{3}{x^2}\right)}{x\left(1-\frac{4}{x}\right)}=\lim_{x\to-\infty}\left(x.\frac{3-\frac{1}{x}+\frac{3}{x^2}}{1-\frac{4}{x}}\right)=-\infty,$$

$$\left(\text{vì }\lim_{x\to-\infty}x=-\infty\text{ và }\lim_{x\to-\infty}\frac{3-\frac{1}{x}+\frac{3}{x^2}}{1-\frac{4}{x}}=3\right).$$

10) $$B=\lim_{x\to-\infty}\frac{2x^3-2x+3}{5-x}=\lim_{x\to-\infty}\frac{x^3\left(2-\frac{2}{x^2}+\frac{3}{x^3}\right)}{x\left(\frac{5}{x}-1\right)}=\lim_{x\to-\infty}\left(x^2.\frac{2-\frac{2}{x^2}+\frac{3}{x^3}}{\frac{5}{x}-1}\right)=-\infty,$$

$$\left(\text{vì }\lim_{x\to-\infty}x^2=+\infty\text{ và }\lim_{x\to-\infty}\frac{2-\frac{2}{x^2}+\frac{3}{x^3}}{\frac{5}{x}-1}=-2\right).$$

Bài 3. 1) $$C=\lim_{x\to+\infty}\left(\sqrt{x^2-3x+2}-x+10\right)=10+\lim_{x\to+\infty}\left(\sqrt{x^2-3x+2}-x\right)$$

$$=10+\lim_{x\to+\infty}\frac{-3x+2}{\sqrt{x^2-3x+2}+x}=10+\lim_{x\to+\infty}\frac{-3x+2}{\sqrt{x^2-3x+2}+x}$$

$$=10+\lim_{x\to+\infty}\frac{-3+\frac{2}{x}}{\sqrt{1-\frac{3}{x}+\frac{2}{x^2}}+1}=10+\frac{-3}{2}=\frac{17}{2}.$$

2) $C=\lim\limits_{x\to+\infty}\dfrac{\sqrt{2x^4+x^2-1}}{1-2x}=\lim\limits_{x\to+\infty}\dfrac{x^2\sqrt{2+\dfrac{1}{x^2}-\dfrac{1}{x^4}}}{x\left(\dfrac{1}{x}-2\right)}=\lim\limits_{x\to+\infty}x\dfrac{\sqrt{2+\dfrac{1}{x^2}-\dfrac{1}{x^4}}}{\dfrac{1}{x}-2}=-\infty$,

$\left(\text{vì }\lim\limits_{x\to+\infty}x=+\infty\text{ và }\lim\limits_{x\to+\infty}\dfrac{\sqrt{2+\dfrac{1}{x^2}-\dfrac{1}{x^4}}}{\left(\dfrac{1}{x}-2\right)}=-\dfrac{\sqrt{2}}{2}<0\right)$

3) $C=\lim\limits_{x\to-\infty}\sqrt{4x^2-4x+1}+2x+13\ =13+\lim\limits_{x\to-\infty}\sqrt{4x^2-4x+1}+2x$

$13+\lim\limits_{x\to-\infty}\dfrac{4x^2-4x+1-4x^2}{\sqrt{4x^2-4x+1}-2x}\ =13+\lim\limits_{x\to-\infty}\dfrac{-4x+1}{\sqrt{x^2\left(4-\dfrac{4}{x}+\dfrac{1}{x^2}\right)}-2x}$

$=13+\lim\limits_{x\to-\infty}\dfrac{-x\left(4+\dfrac{1}{x}\right)}{|x|\sqrt{\left(4-\dfrac{4}{x}+\dfrac{1}{x^2}\right)}-2x}\ =13+\lim\limits_{x\to-\infty}\dfrac{-x\left(4+\dfrac{1}{x}\right)}{-x\sqrt{\left(4-\dfrac{4}{x}+\dfrac{1}{x^2}\right)}-2x}$

$=13+\lim\limits_{x\to-\infty}\dfrac{4+\dfrac{1}{x}}{\sqrt{\left(4-\dfrac{4}{x}+\dfrac{1}{x^2}\right)}+2}=14$

4) $C=\lim\limits_{x\to-\infty}\sqrt{x^2+x}+x+5\ =5+\lim\limits_{x\to-\infty}\left(-x\sqrt{1+\dfrac{1}{x}}+x\right)=5+\lim\limits_{x\to-\infty}\left[x\left(1-\sqrt{1+\dfrac{1}{x}}\right)\right]$

$=5+\lim\limits_{x\to-\infty}\left[x\left(\dfrac{1-1-\dfrac{1}{x}}{1+\sqrt{1+\dfrac{1}{x}}}\right)\right]=5+\lim\limits_{x\to-\infty}\left(\dfrac{-1}{1+\sqrt{1+\dfrac{1}{x}}}\right)=\dfrac{9}{2}$.

5) $C=\lim\limits_{x\to-\infty}\sqrt{2x^2+1}+x\ =\lim\limits_{x\to-\infty}\left(-x\sqrt{2+\dfrac{1}{x^2}}+x\right)=\lim\limits_{x\to-\infty}\left[x\left(1-\sqrt{2+\dfrac{1}{x^2}}\right)\right]=+\infty$,

$\left(\text{vì }\lim\limits_{x\to-\infty}\left(1-\sqrt{2+\dfrac{1}{x^2}}\right)=1-\sqrt{2}<0\text{ và }\lim\limits_{x\to-\infty}x=-\infty\right)$.

6) $C=\lim\limits_{x\to+\infty}\sqrt{x^2-4x}-x+2021\ =2021+\lim\limits_{x\to+\infty}\left(x\sqrt{1-\dfrac{4}{x}}-x\right)=2021+\lim\limits_{x\to+\infty}\left[x\left(\sqrt{1-\dfrac{4}{x}}-1\right)\right]$

$=2021+\lim\limits_{x\to+\infty}\left[x\left(\dfrac{1-\dfrac{4}{x}-1}{\sqrt{1-\dfrac{4}{x}}+1}\right)\right]=2021+\lim\limits_{x\to+\infty}\left(\dfrac{-4}{\sqrt{1-\dfrac{4}{x}}+1}\right)=2021-\dfrac{4}{2}=2019$.

7) $C=\lim\limits_{x\to-\infty}\sqrt{x^2+x}-\sqrt{x^2+1}=\lim\limits_{x\to-\infty}\dfrac{x-1}{\sqrt{x^2+x}+\sqrt{x^2+1}}$

$=\lim\limits_{x\to-\infty}\dfrac{x-1}{|x|\sqrt{1+\dfrac{1}{x}}+|x|\sqrt{1+\dfrac{1}{x^2}}}=\lim\limits_{x\to-\infty}\dfrac{x-1}{-x\sqrt{1+\dfrac{1}{x}}-x\sqrt{1+\dfrac{1}{x^2}}}$

$\lim\limits_{x\to-\infty}\dfrac{1-\dfrac{1}{x}}{-\sqrt{1+\dfrac{1}{x}}-\sqrt{1+\dfrac{1}{x^2}}}=-\dfrac{1}{2}$

8) $C=\lim\limits_{x\to-\infty}\dfrac{2|x|+3}{\sqrt{x^2+x+5}}=\lim\limits_{x\to-\infty}\dfrac{2|x|+3}{-x\sqrt{1+\dfrac{1}{x}+\dfrac{5}{x^2}}}=\lim\limits_{x\to-\infty}\dfrac{\dfrac{2|x|}{x}+\dfrac{3}{x}}{-\sqrt{1+\dfrac{1}{x}+\dfrac{5}{x^2}}}=\dfrac{2+0}{-\sqrt{1+0+0}}=-2.$

9) $C=\lim\limits_{x\to+\infty}\sqrt{4x^4+3x^2+1}-2x^2=\lim\limits_{x\to+\infty}\left(x^2\sqrt{4+\dfrac{3}{x^2}+\dfrac{1}{x^4}}-2x^2\right)$

$=\lim\limits_{x\to+\infty}\left(x^2\dfrac{4+\dfrac{3}{x^2}+\dfrac{1}{x^4}-4}{\sqrt{4+\dfrac{3}{x^2}+\dfrac{1}{x^4}}+2}\right)=\lim\limits_{x\to+\infty}\dfrac{3+\dfrac{1}{x^2}}{\sqrt{4+\dfrac{3}{x^2}+\dfrac{1}{x^4}}+2}=\dfrac{3}{4}.$

10) $C=\lim\limits_{x\to-\infty}\dfrac{\sqrt{x^2+x}+2x}{2x+3}=\lim\limits_{x\to-\infty}\dfrac{-x\sqrt{1+\dfrac{1}{x}}+2x}{2x+3}$

$=\lim\limits_{x\to-\infty}\dfrac{x\left(-\sqrt{1+\dfrac{1}{x}}+2\right)}{x\left(2+\dfrac{3}{x}\right)}+\lim\limits_{x\to-\infty}\dfrac{-\sqrt{1+\dfrac{1}{x}}+2}{2+\dfrac{3}{x}}=\dfrac{-1+2}{2}=\dfrac{1}{2}.$

11) $C=\lim\limits_{x\to+\infty}\sqrt{x^2+x+1}-x+1=1+\lim\limits_{x\to+\infty}\left(x\sqrt{1+\dfrac{1}{x}+\dfrac{1}{x^2}}-x\right)$

$=1+\lim\limits_{x\to+\infty}\left[x\left(\sqrt{1+\dfrac{1}{x}+\dfrac{1}{x^2}}-1\right)\right]=1+\lim\limits_{x\to+\infty}\left[x\left(\dfrac{1+\dfrac{1}{x}+\dfrac{1}{x^2}-1}{\sqrt{1+\dfrac{1}{x}+\dfrac{1}{x^2}}+1}\right)\right]$

$=1+\lim\limits_{x\to+\infty}\left(\dfrac{1+\dfrac{1}{x}}{\sqrt{1+\dfrac{1}{x}+\dfrac{1}{x^2}}+1}\right)=1+\dfrac{1}{2}=\dfrac{3}{2}.$

12) $C=\lim\limits_{x\to-\infty}\dfrac{|x|-\sqrt{x^2+x}}{x+10}=\lim\limits_{x\to-\infty}\dfrac{|x|-x\sqrt{1+\dfrac{1}{x}}}{x+10}=\lim\limits_{x\to-\infty}\dfrac{\dfrac{|x|}{x}-\sqrt{1+\dfrac{1}{x}}}{1+\dfrac{10}{x}}=\dfrac{-1-1}{1}=-2$.

13) $C=\lim\limits_{x\to-\infty}\sqrt{4x^2-9x-21}-\sqrt{4x^2-7x+13}=\lim\limits_{x\to-\infty}\left(-x\sqrt{4-\dfrac{9}{x}-\dfrac{21}{x^2}}+x\sqrt{4-\dfrac{7}{x}+\dfrac{13}{x^2}}\right)$

$=\lim\limits_{x\to-\infty}\left[x\left(\dfrac{-4+\dfrac{9}{x}+\dfrac{21}{x^2}+4-\dfrac{7}{x}+\dfrac{13}{x^2}}{\sqrt{4-\dfrac{9}{x}-\dfrac{21}{x^2}}+\sqrt{4-\dfrac{7}{x}+\dfrac{13}{x^2}}}\right)\right]=\lim\limits_{x\to-\infty}\dfrac{2+\dfrac{34}{x}}{\sqrt{4-\dfrac{9}{x}-\dfrac{21}{x^2}}+\sqrt{4-\dfrac{7}{x}+\dfrac{13}{x^2}}}=\dfrac{2}{2+2}=\dfrac{1}{2}$.

14) $C=\lim\limits_{x\to-\infty}\dfrac{4x^2|x|-3x^2+7x-1}{2x+1^{\ 2}.\sqrt{x^2+3x}}$

$=\lim\limits_{x\to-\infty}\dfrac{x^3\left(\dfrac{4|x|}{x}-\dfrac{3}{x}+\dfrac{7}{x^2}-\dfrac{1}{x^3}\right)}{x^2\left(2+\dfrac{1}{x}\right)^2.-x\sqrt{1+\dfrac{3}{x}}}=\lim\limits_{x\to-\infty}\dfrac{\left(\dfrac{4|x|}{x}-\dfrac{3}{x}+\dfrac{7}{x^2}-\dfrac{1}{x^3}\right)}{-\left(2+\dfrac{1}{x}\right)^2.\sqrt{1+\dfrac{3}{x}}}=\dfrac{-4}{-2^2.1}=1$

15) $C=\lim\limits_{x\to-\infty}\sqrt{4x^2-4x+1}+2x+3=3+\lim\limits_{x\to-\infty}\left(-x\sqrt{4-\dfrac{4}{x}+\dfrac{1}{x^2}}+2x\right)$

$=3+\lim\limits_{x\to-\infty}\left[x\left(\dfrac{4-4+\dfrac{4}{x}-\dfrac{1}{x^2}}{\sqrt{4-\dfrac{4}{x}+\dfrac{1}{x^2}}+2}\right)\right]=3+\lim\limits_{x\to-\infty}\dfrac{4-\dfrac{1}{x}}{\sqrt{4-\dfrac{4}{x}+\dfrac{1}{x^2}}+2}=3+\dfrac{4}{4}=4$.

16) $C=\lim\limits_{x\to-\infty}\left[x-1\sqrt{\dfrac{3-x}{5-3x-x^3}}\right]$

$=\lim\limits_{x\to-\infty}\left[x\left(1-\dfrac{1}{x}\right)\sqrt{\dfrac{\dfrac{3}{x}-1}{x^2\left(\dfrac{5}{x^3}-\dfrac{3}{x^2}-1\right)}}\right]=\lim\limits_{x\to-\infty}\left[-\left(1-\dfrac{1}{x}\right)\sqrt{\dfrac{\dfrac{3}{x}-1}{\left(\dfrac{5}{x^2}-\dfrac{3}{x}-1\right)}}\right]=-1.\sqrt{\dfrac{0-1}{0-0-1}}=-1$.

17) $C=\lim\limits_{x\to+\infty}\sqrt{16x^2+3x}-4x+5$

$=5+\lim\limits_{x\to+\infty}\left(x\sqrt{16+\dfrac{3}{x}}-4x\right)=5+\lim\limits_{x\to+\infty}\left[x\left(\sqrt{16+\dfrac{3}{x}}-4\right)\right]$

$=5+\lim\limits_{x\to+\infty}\left[x\left(\dfrac{16+\dfrac{3}{x}-16}{\sqrt{16+\dfrac{3}{x}}+4}\right)\right]=5+\lim\limits_{x\to+\infty}\dfrac{3}{\sqrt{16+\dfrac{3}{x}}+4}=5+\dfrac{3}{4+4}=5+\dfrac{3}{8}=\dfrac{43}{8}$.

18) $C=\lim_{x\to-\infty}\left(x\sqrt{\frac{2x^3+x}{x^5-x^2+3}}\right)=\lim_{x\to-\infty}\left(x\sqrt{\frac{x^3\left(2+\frac{1}{x^2}\right)}{x^5\left(1-\frac{1}{x^3}+\frac{3}{x^5}\right)}}\right)=\lim_{x\to-\infty}\left(-\sqrt{\frac{2+\frac{1}{x^2}}{1-\frac{1}{x^3}+\frac{3}{x^5}}}=\right)-\sqrt{2}.$

19) $C=\lim_{x\to+\infty}\ x-3-\sqrt{x^2-x+1}\ =-3+\lim_{x\to-\infty}\left(x-x\sqrt{1-\frac{1}{x}+\frac{1}{x^2}}\right)$

$=-3+\lim_{x\to-\infty}\left[x\left(1-\sqrt{1-\frac{1}{x}+\frac{1}{x^2}}\right)\right]=-3+\lim_{x\to-\infty}\left[x\left(\frac{1-1+\frac{1}{x}-\frac{1}{x^2}}{1+\sqrt{1-\frac{1}{x}+\frac{1}{x^2}}}\right)\right]$

$=-3+\lim_{x\to-\infty}\frac{1-\frac{1}{x}}{1+\sqrt{1-\frac{1}{x}+\frac{1}{x^2}}}=-3+\frac{1}{2}=-\frac{5}{2}.$

Bài 4.

1) $\lim_{x\to-\infty}\left[x.\sqrt{\frac{2x^3+x}{x^5-x^2+3}}\right]=\lim_{x\to-\infty}\left[x.\frac{-x}{x^2}.\sqrt{\frac{2x^3+x}{x^5-x^2+3}}\right]=\lim_{x\to-\infty}\left[-1\ .\sqrt{\frac{2x+\frac{1}{x}}{x-\frac{1}{x^2}+\frac{3}{x^4}}}\right]=-\sqrt{2}.$

2) $\lim_{x\to-\infty}\frac{2|x|+3}{\sqrt{x^2+x+5}}=\lim_{x\to-\infty}\frac{-2x+3}{-x\sqrt{1+\frac{1}{x}+\frac{5}{x^2}}}=\lim_{x\to-\infty}\frac{-2+\frac{3}{x}}{-\sqrt{1+\frac{1}{x}+\frac{5}{x^2}}}=2\ .$

3) $\lim_{x\to+\infty}\frac{\sqrt{x^2+x+2}+3x+1}{\sqrt{4x^2+1}+1-x}=\lim_{x\to+\infty}\frac{x\sqrt{1+\frac{1}{x}+\frac{2}{x^2}}+3x+1}{x\sqrt{4+\frac{1}{x^2}}+1-x}=\lim_{x\to+\infty}\frac{\sqrt{1+\frac{1}{x}+\frac{2}{x^2}}+3+\frac{1}{x}}{\sqrt{4+\frac{1}{x^2}}+\frac{1}{x}-1}=\frac{1+3}{2-1}=4.$

4) $\lim_{x\to-\infty}\frac{\sqrt{2x^4-x^2}-x\sqrt{x^2+3}}{x\ 5-2x}=\lim_{x\to-\infty}\frac{x^2\sqrt{2-\frac{1}{x^2}}+x^2\sqrt{1+\frac{3}{x}}}{x^2\left(\frac{5}{x}-2\right)}=\lim_{x\to-\infty}\frac{\sqrt{2-\frac{1}{x^2}}+\sqrt{1+\frac{3}{x}}}{\left(\frac{5}{x}-2\right)}=-\frac{\sqrt{2}+1}{2}.$

5) $\lim_{x\to+\infty}\frac{2x-1+3\sqrt{4x^2+x-5}}{1-3x+2\sqrt{9x^2-x+10}}=\lim_{x\to+\infty}\frac{2x-1+3x\sqrt{4+\frac{1}{x}-\frac{5}{x^2}}}{1-3x+2x\sqrt{9-\frac{1}{x}+\frac{10}{x^2}}}=\lim_{x\to+\infty}\frac{2-\frac{1}{x}+3\sqrt{4+\frac{1}{x}-\frac{5}{x^2}}}{\frac{1}{x}-3+2\sqrt{9-\frac{1}{x}+\frac{10}{x^2}}}=\frac{8}{3}.$

6) $\lim_{x\to-\infty}\frac{3\sqrt{x^2-1}-\sqrt[3]{1-8x^3}}{6x+9}=\lim_{x\to-\infty}\frac{-3x\sqrt{1-\frac{1}{x^2}}-x\sqrt[3]{\frac{1}{x^3}-8}}{6x+9}=\lim_{x\to-\infty}\frac{-3\sqrt{1-\frac{1}{x^2}}-\sqrt[3]{\frac{1}{x^3}-8}}{6+\frac{9}{x}}=-\frac{1}{6}.$

7) $\lim\limits_{x\to-\infty}\dfrac{2x-1\ \sqrt{x^2-3}}{x-5x^2}=\lim\limits_{x\to-\infty}\dfrac{2x-1\ \ -x\sqrt{1-\frac{3}{x}}}{x-5x^2}=\lim\limits_{x\to-\infty}\dfrac{\left(2-\frac{1}{x}\right)-1\sqrt{1-\frac{3}{x}}}{\frac{1}{x}-5}=\dfrac{2}{5}$.

8) $\lim\limits_{x\to-\infty}\dfrac{x-\sqrt{4x^2+3x-1}}{\sqrt{9x^2-x+3}-5x-3}=\lim\limits_{x\to-\infty}\dfrac{x+x\sqrt{4+\frac{3}{x}-\frac{1}{x^2}}}{-x\sqrt{9-\frac{1}{x}+\frac{3}{x^2}}-5x-3}=\lim\limits_{x\to-\infty}\dfrac{1+\sqrt{4+\frac{3}{x}-\frac{1}{x^2}}}{-\sqrt{9-\frac{1}{x}+\frac{3}{x^2}}-5-\frac{3}{x}}=-\dfrac{1}{4}$.

9) $\lim\limits_{x\to+\infty}\dfrac{2x+1}{\sqrt{x^2-x}+\sqrt{x^2+x}}=\lim\limits_{x\to+\infty}\dfrac{2x+1}{x\sqrt{1-\frac{1}{x}}+x\sqrt{1+\frac{1}{x}}}=\lim\limits_{x\to+\infty}\dfrac{2+\frac{1}{x}}{\sqrt{1-\frac{1}{x}}+\sqrt{1+\frac{1}{x}}}=\dfrac{2}{2}=1$.

10) $\lim\limits_{x\to-\infty}\dfrac{8x+3}{6x+\sqrt{4x^2+x+3}}=\lim\limits_{x\to-\infty}\dfrac{8x+3}{6x-x\sqrt{4+\frac{1}{x}+\frac{3}{x^2}}}=\lim\limits_{x\to-\infty}\dfrac{8+\frac{3}{x}}{6-\sqrt{4+\frac{1}{x}+\frac{3}{x^2}}}=2$.

11) $\lim\limits_{x\to+\infty}\dfrac{\sqrt{x^2+1}-7x+2}{\sqrt{x^2+3x+2}+5x+3}=\lim\limits_{x\to+\infty}\dfrac{x\sqrt{1+\frac{1}{x^2}}-7x+2}{x\sqrt{1+\frac{3}{x}+\frac{2}{x^2}}+5x+3}=\lim\limits_{x\to+\infty}\dfrac{\sqrt{1+\frac{1}{x^2}}-7+\frac{2}{x}}{x\sqrt{1+\frac{3}{x}+\frac{2}{x^2}}+5+\frac{3}{x}}=-1$.

12) $\lim\limits_{x\to-\infty}\dfrac{x+2\sqrt{1-2x}}{1-x}=\lim\limits_{x\to-\infty}\dfrac{x-2x\sqrt{\frac{1}{x^2}-\frac{2}{x}}}{1-x}=\lim\limits_{x\to-\infty}\dfrac{1-2\sqrt{\frac{1}{x^2}-\frac{2}{x}}}{\frac{1}{x}-1}=-1$.

Bài 5.

1) $\lim\limits_{x\to-\infty}\ \sqrt{x^2+x}-x\ =\lim\limits_{x\to-\infty}\left(-x\sqrt{1+\frac{1}{x}}-x\right)=\lim\limits_{x\to-\infty}\left[x\left(-\sqrt{1+\frac{1}{x}}-1\right)\right]=+\infty$.

Vì $\lim\limits_{x\to-\infty}x=-\infty$ và $\lim\limits_{x\to-\infty}\left(-\sqrt{1+\frac{1}{x}}-1\right)=-2$.

2) $\lim\limits_{x\to+\infty}\ \sqrt{x^2-4x}-x\ =\lim\limits_{x\to+\infty}\dfrac{x^2-4x-x^2}{\sqrt{x^2-4x}+x}=\lim\limits_{x\to+\infty}\dfrac{-4x}{x\sqrt{1-\frac{4}{x}}+1}=-2$.

3) $\lim\limits_{x\to+\infty}\ \sqrt{x+2}-\sqrt{x-2}\ =\lim\limits_{x\to+\infty}\dfrac{x+2-\ x-2}{\sqrt{x+2}+\sqrt{x-2}}=\lim\limits_{x\to+\infty}\dfrac{4}{\sqrt{x}\left(\sqrt{1+\frac{2}{x}}+\sqrt{1-\frac{2}{x}}\right)}=0$.

Vì $\lim\limits_{x\to+\infty}\dfrac{4}{\sqrt{x}}=0$ và $\lim\limits_{x\to+\infty}\dfrac{1}{\left(\sqrt{1+\frac{2}{x}}+\sqrt{1-\frac{2}{x}}\right)}=\dfrac{1}{2}$.

4) $\lim\limits_{x\to-\infty}\ \sqrt{x^2+x}-\sqrt{x^2+1}\ =\lim\limits_{x\to-\infty}\frac{x^2+x-\ x^2+1}{\sqrt{x^2+x}+\sqrt{x^2+1}}=\lim\limits_{x\to-\infty}\frac{x-1}{-x\sqrt{1+\frac{1}{x}}-x\sqrt{1+\frac{1}{x^2}}}$

$=\lim\limits_{x\to-\infty}\frac{1-\frac{1}{x}}{-\sqrt{1+\frac{1}{x}}-\sqrt{1+\frac{1}{x^2}}}=-\frac{1}{2}.$

5) $\lim\limits_{x\to+\infty}\ \sqrt{x^2+4x+1}-x-2\ =\lim\limits_{x\to+\infty}\frac{x^2+4x+1-\ x+2\ ^2}{\sqrt{x^2+4x+1}+x+2}=\lim\limits_{x\to+\infty}\frac{-3}{\left(x\sqrt{1+\frac{4}{x}+\frac{1}{x^2}}+x+2\right)}$

$=\lim\limits_{x\to+\infty}\frac{-3}{x\left(\sqrt{1+\frac{4}{x}+\frac{1}{x^2}}+1+\frac{2}{x}\right)}=0.$

6) $\lim\limits_{x\to-\infty}\ \sqrt{x^2-3x+5}+x+1\ =\lim\limits_{x\to-\infty}\frac{x^2-3x+5-\ x+1\ ^2}{\sqrt{x^2-3x+5}-x-1}=\lim\limits_{x\to-\infty}\frac{-5x+4}{\left(-x\sqrt{1-\frac{3}{x}+\frac{5}{x^2}}-x-1\right)}$

$=\lim\limits_{x\to-\infty}\frac{-5+\frac{4}{x}}{\left(-\sqrt{1-\frac{3}{x}+\frac{5}{x^2}}-1-\frac{1}{x}\right)}=\frac{5}{2}.$

7) $\lim\limits_{x\to+\infty}\ \sqrt[3]{27x^3-x^2}-3x\ =\lim\limits_{x\to+\infty}\frac{27x^3-x^2-27x^3}{\sqrt[3]{27x^3-x^2}\ ^2+3x\sqrt[3]{27x^3-x^2}+9x^2}$

$=\lim\limits_{x\to+\infty}\frac{-x^2}{x^2\left(\sqrt[3]{27-\frac{1}{x}}\right)^2+3x^2\sqrt[3]{27-\frac{1}{x}}+9x^2}=\lim\limits_{x\to+\infty}\frac{-1}{\left(\sqrt[3]{27-\frac{1}{x}}\right)^2+3\sqrt[3]{27-\frac{1}{x}}+9}=-\frac{1}{27}.$

8) $\lim\limits_{x\to+\infty}\ 2x-\sqrt{4x^2+2x-1}\ =\lim\limits_{x\to+\infty}\frac{4x^2-4x^2-2x+1}{2x+\sqrt{4x^2+2x-1}}=\lim\limits_{x\to+\infty}\frac{-2x+1}{2x+x\sqrt{4+\frac{2}{x}-\frac{1}{x^2}}}$

$=\lim\limits_{x\to+\infty}\frac{-2+\frac{1}{x}}{2+\sqrt{4+\frac{2}{x}-\frac{1}{x^2}}}=-\frac{1}{2}.$

9) $\lim\limits_{x\to+\infty}\ 2x-3-\sqrt{4x^2+4x+3}\ =\lim\limits_{x\to+\infty}\frac{2x-3\ ^2-4x^2-4x-3}{2x-3+\sqrt{4x^2+4x+3}}=\lim\limits_{x\to+\infty}\frac{-16x+6}{2x-3+\sqrt{4x^2+4x+3}}$

$$=\lim_{x\to+\infty}\frac{-16x+6}{\left(2x-3+x\sqrt{4+\frac{4}{x}+\frac{3}{x^2}}\right)}=\lim_{x\to+\infty}\frac{-16+\frac{6}{x}}{\left(2-\frac{3}{x}+\sqrt{4+\frac{4}{x}+\frac{3}{x^2}}\right)}=-4$$

10) $$\lim_{x\to-\infty}\left(\sqrt{4x^4+3x^2+1}-2x^2\right)=\lim_{x\to-\infty}\frac{4x^4+3x^2+1-4x^4}{\sqrt{4x^4+3x^2+1}+2x^2}=\lim_{x\to-\infty}\frac{3x^2+1}{x^2\sqrt{4+\frac{3}{x^2}+\frac{1}{x^4}}+2x^2}$$

$$=\lim_{x\to-\infty}\frac{3+\frac{1}{x^2}}{\sqrt{4+\frac{3}{x^2}+\frac{1}{x^4}}+2}=\frac{3}{4}.$$

11) $$\lim_{x\to+\infty}\left(\sqrt{4x^2+3x+1}-2x+4\right)=\lim_{x\to+\infty}\frac{4x^2+3x+1-(2x-4)^2}{\sqrt{4x^2+3x+1}+2x-4}=\lim_{x\to+\infty}\frac{19x-15}{\left(x\sqrt{4+\frac{3}{x}+\frac{1}{x^2}}+2x-4\right)}$$

$$=\lim_{x\to+\infty}\frac{19-\frac{15}{x}}{\left(\sqrt{4+\frac{3}{x}+\frac{1}{x^2}}+2-\frac{4}{x}\right)}=\frac{19}{4}.$$

12) $$\lim_{x\to-\infty}\left(\sqrt{4x^2-4x+1}+2x+3\right)=\lim_{x\to-\infty}\frac{4x^2-4x+1-(2x+3)^2}{\sqrt{4x^2-4x+1}-2x-3}=\lim_{x\to-\infty}\frac{-16x-8}{\left(-x\sqrt{4-\frac{4}{x}+\frac{1}{x^2}}-2x-3\right)}$$

$$=\lim_{x\to-\infty}\frac{-16-\frac{8}{x}}{\left(-\sqrt{4-\frac{4}{x}+\frac{1}{x^2}}-2-\frac{3}{x}\right)}=4.$$

13) $$\lim_{x\to+\infty}\frac{x+\sqrt[3]{4x^2-x^3}}{2x-\sqrt{4x^2-3x}}=\lim_{x\to+\infty}\left[\frac{x^3+4x^2-x^3}{4x^2-4x^2+3x}\cdot\frac{2x+\sqrt{4x^2-3x}}{x^2-x\sqrt[3]{4x^2-x^3}+\left(\sqrt[3]{4x^2-x^3}\right)^2}\right]$$

$$=\lim_{x\to+\infty}\left[\frac{4}{3}\cdot\frac{2+\sqrt{4-\frac{3}{x}}}{1-\sqrt[3]{\frac{4}{x}-1}+\left(\sqrt[3]{\frac{4}{x}-1}\right)^2}\right]=\frac{16}{9}.$$

14) $$\lim_{x\to+\infty}\left(\sqrt[3]{8x^3+1}-2x+1\right)=\lim_{x\to+\infty}\frac{8x^3+1-(2x-1)^3}{\left(\sqrt[3]{8x^3+1}\right)^2+(2x-1)\sqrt[3]{8x^3+1}+(2x-1)^2}$$

$$=\lim_{x\to+\infty}\frac{12x^2-6x+2}{\left(\sqrt[3]{8x^3+1}\right)^2+\left(2x-1\right)\sqrt[3]{8x^3+1}+\left(2x-1\right)^2}=\lim_{x\to+\infty}\frac{12-\frac{6}{x}+\frac{2}{x^2}}{\left(\sqrt[3]{8+\frac{1}{x^3}}\right)^2+\left(2-\frac{1}{x}\right)\sqrt[3]{8+\frac{1}{x^3}}+\left(2-\frac{1}{x}\right)^2}=1.$$

◆Dạng 4. Giới hạn một bên $x\to x_0^+$ hoặc $x\to x_0^-$.

Phương pháp giải:

- Sử dụng các định lý về giới hạn hàm số

Chú ý: $x\to x_0^+\Rightarrow x>x_0\Rightarrow x-x_0>0$

$x\to x_0^-\Rightarrow x<x_0\Rightarrow x-x_0<0$

❶ VÍ DỤ

Ví dụ 1. Tính giới hạn $A=\lim\limits_{x\to1^+}\frac{2x-3}{x-1}$. **Đs:** $-\infty$.

Lời giải

Vì $\begin{cases}\lim\limits_{x\to1^+}\left(2x-3\right)=-1<0\\ \lim\limits_{x\to1^+}\left(x-1\right)=0\\ x\to1^+\Rightarrow x>1\Rightarrow x-1>0\end{cases}\Rightarrow A=\lim\limits_{x\to1^+}\frac{2x-3}{x-1}=-\infty.$

Ví dụ 2. Tính giới hạn $A=\lim\limits_{x\to2^+}\frac{x-15}{x-2}$. **Đs:** $-\infty$.

Lời giải

Vì $\begin{cases}\lim\limits_{x\to2^+}\left(x-15\right)=-13<0\\ \lim\limits_{x\to2^+}\left(x-2\right)=0\\ x\to2^+\Rightarrow x>2\Rightarrow x-2>0\end{cases}\Rightarrow A=\lim\limits_{x\to2^+}\frac{x-15}{x-2}=-\infty.$

Ví dụ 3. Tính giới hạn $A=\lim\limits_{x\to3^-}\frac{2-x}{3-x}$. **Đs:** $-\infty$.

Lời giải

Vì $\begin{cases}\lim\limits_{x\to3^-}\left(2-x\right)=-1<0\\ \lim\limits_{x\to3^-}\left(3-x\right)=0\\ x\to3^-\Rightarrow x<3\Rightarrow 3-x>0\end{cases}\Rightarrow A=\lim\limits_{x\to3^-}\frac{2-x}{3-x}=-\infty.$

Ví dụ 4. Tính giới hạn $A=\lim\limits_{x\to2^+}\frac{x+1}{2x-4}$. **Đs:** $+\infty$.

Lời giải

Vì $\begin{cases} \lim\limits_{x\to 2^+}(x+1)=3>0 \\ \lim\limits_{x\to 2^+}(2x-4)=0 \\ x\to 2^+ \Rightarrow x>2 \Rightarrow 2x-4>0 \end{cases} \Rightarrow A=\lim\limits_{x\to 2^+}\dfrac{x+1}{2x-4}=+\infty$.

Ví dụ 5. Tính giới hạn $A=\lim\limits_{x\to 4^-}\dfrac{x-5}{(x-4)^2}$. **Đs:** $-\infty$.

Lời giải

Vì $\begin{cases} \lim\limits_{x\to 4^-}(x-5)=-1<0 \\ \lim\limits_{x\to 4^-}(x-4)^2=0 \\ x\to 4^- \Rightarrow (x-4)^2>0 \end{cases} \Rightarrow A=\lim\limits_{x\to 4^-}\dfrac{x-5}{(x-4)^2}=-\infty$.

Ví dụ 6. Tính giới hạn $A=\lim\limits_{x\to 3^-}\dfrac{3x-8}{(3-x)^2}$. **Đs:** $+\infty$.

Lời giải

Vì $\begin{cases} \lim\limits_{x\to 3^-}(3x-8)=1>0 \\ \lim\limits_{x\to 3^-}(3-x)^2=0 \\ x\to 3^- \Rightarrow (3-x)^2>0 \end{cases} \Rightarrow A=\lim\limits_{x\to 3^-}\dfrac{3x-8}{(3-x)^2}=+\infty$.

Ví dụ 7. Tính giới hạn $A=\lim\limits_{x\to(-3)^+}\dfrac{2x^2+5x-3}{(x+3)^2}$. **Đs:** $-\infty$.

Lời giải

Ta có $\lim\limits_{x\to(-3)^+}\dfrac{2x^2+5x-3}{(x+3)^2}=\lim\limits_{x\to(-3)^+}\dfrac{(2x-1)(x+3)}{(x+3)^2}=\lim\limits_{x\to(-3)^+}\dfrac{2x-1}{x+3}$

Vì $\begin{cases} \lim\limits_{x\to(-3)^+}(2x-1)=-7<0 \\ \lim\limits_{x\to(-3)^+}(x+3)=0 \\ x\to(-3)^+ \Rightarrow x>-3 \Rightarrow x+3>0 \end{cases} \Rightarrow A=\lim\limits_{x\to(-3)^+}\dfrac{2x^2+5x-3}{(x+3)^2}=-\infty$.

Ví dụ 8. Tính giới hạn $A=\lim\limits_{x\to 2^-}\left(\dfrac{1}{x-2}-\dfrac{1}{x^2-4}\right)$. **Đs:** $-\infty$.

Lời giải

Ta có: $A=\lim\limits_{x\to 2^-}\left(\dfrac{1}{x-2}-\dfrac{1}{x^2-4}\right)=\lim\limits_{x\to 2^-}\dfrac{x+1}{(x-2)(x+2)}$

Vì $\begin{cases} \lim\limits_{x\to 2^-}(x+1)=3>0 \\ \lim\limits_{x\to 2^-}\left[(x-2)(x+2)\right]=0 \\ x\to 2^- \Rightarrow x<2 \Rightarrow (x-2)(x+2)<0 \end{cases}$ $\Rightarrow A=\lim\limits_{x\to 2^-}\left(\dfrac{1}{x-2}-\dfrac{1}{x^2-4}\right)=-\infty$.

Ví dụ 9. Tính giới hạn $B=\lim\limits_{x\to 2^-}\dfrac{|2-x|}{2x^2-5x+2}$. **Đs:** $-\dfrac{1}{3}$.

Lời giải

Vì $x\to 2^- \Rightarrow x<2 \Rightarrow |2-x|=2-x$

Do đó $B=\lim\limits_{x\to 2^-}\dfrac{2-x}{(x-2)(2x-1)}=\lim\limits_{x\to 2^-}\dfrac{-1}{2x-1}=-\dfrac{1}{3}$.

Ví dụ 10. Tính giới hạn $B=\lim\limits_{x\to 3^+}\dfrac{|x-3|}{5x-15}$. **Đs:** $\dfrac{1}{5}$.

Lời giải

Vì $x\to 3^+ \Rightarrow x>3 \Rightarrow |x-3|=x-3$

Do đó $B=\lim\limits_{x\to 3^-}\dfrac{x-3}{5(x-3)}=\lim\limits_{x\to 3^-}\dfrac{1}{5}=\dfrac{1}{5}$.

❷ BÀI TẬP ÁP DỤNG

Bài 1. Tính các giới hạn sau:

1) $A=\lim\limits_{x\to 1^-}\dfrac{|x-1|}{2x^3+x-3}$. **Đs:** $-\dfrac{1}{7}$.

2) $B=\lim\limits_{x\to 2}\dfrac{|x-2|}{x-2}$. **Đs:** Không tồn tại.

3) $C=\lim\limits_{x\to 3}\dfrac{|x^2-9|}{x-3}$. **Đs:** Không tồn tại.

Bài 2. Tính các giới hạn sau:

1) $C=\lim\limits_{x\to 1^-}\dfrac{2x^2-2x+|x-1|\sqrt{x+3}}{x^2-2x+1}$. **Đs:** $\dfrac{7}{4}$.

2) $C=\lim\limits_{x\to 2^-}\dfrac{|x-2|}{\sqrt{x-1}-1}$. **Đs:** -2.

3) $D=\lim\limits_{x\to 3^-}\dfrac{\sqrt{x^2-7x+12}}{\sqrt{9-x^2}}$. **Đs:** $\dfrac{1}{\sqrt{6}}$.

4) $D=\lim\limits_{x\to 2^-}\dfrac{\sqrt{x^2-5x+6}}{\sqrt{4-x^2}}$. **Đs:** $\dfrac{1}{2}$.

5) $D=\lim\limits_{x\to 1^-}\dfrac{\sqrt{1-x}+x-1}{\sqrt{x^2-x^3}}$. **Đs:** 1.

6) $D=\lim\limits_{x\to 1^+}(1-x)\sqrt{\dfrac{x+5}{x^3+2x^2-3}}$. **Đs:** 0.

7) $D=\lim\limits_{x\to 1^-}\dfrac{\sqrt{x^3-3x+2}}{x^2-5x+4}$. **Đs:** $\dfrac{\sqrt{3}}{3}$.

Bài 3. 1) Tính giới hạn $C=\lim\limits_{x\to 1}f(x)$ với $f(x)=\begin{cases}5x^4-6x^2-x & khi\ x\ge 1\\ x^3-3x & khi\ x<1\end{cases}$. **Đs:** -2

2) Tính giới hạn $C=\lim\limits_{x\to 1}f(x)$ với $f(x)=\begin{cases}x-3 & khi\ x<1\\ 1-\sqrt{7x^2+2} & khi\ x\ge 1\end{cases}$. **Đs:** -2.

3) Tính giới hạn $C=\lim\limits_{x\to -2}f(x)$ với $f(x)=\begin{cases}\dfrac{3x-2}{x+1} & khi\ x<-2\\ x+10 & khi\ x\ge -2\end{cases}$. **Đs:** 8.

Bài 4. Tìm m để hàm số $f(x)=\begin{cases}\dfrac{x^3+1}{x+1} & khi\ x<-1\\ mx^2-x+m^2 & khi\ x\ge -1\end{cases}$ có giới hạn tại $x=-1$.

Đs: $m=1$ hoặc $m=-2$.

❸ LỜI GIẢI

Bài 1. 1) $A=\lim\limits_{x\to 1^-}\dfrac{|x-1|}{2x^3+x-3}$.

Vì $x\to 1^-\Rightarrow x<1\Rightarrow |x-1|=-(x-1)$.

Do đó $A=\lim\limits_{x\to 1^-}\dfrac{-(x-1)}{(x-1)(2x^2+2x+3)}=\lim\limits_{x\to 1^-}\dfrac{-1}{2x^2+2x+3}=-\dfrac{1}{7}$.

2) $B=\lim\limits_{x\to 2}\dfrac{|x-2|}{x-2}$.

+) Vì $x\to 2^-\Rightarrow x<2\Rightarrow |x-2|=-(x-2)$ nên $\lim\limits_{x\to 2^-}\dfrac{-(x-2)}{x-2}=\lim\limits_{x\to 2^-}(-1)=-1$.

+) Vì $x\to 2^+\Rightarrow x>2\Rightarrow |x-2|=x-2$ nên $\lim\limits_{x\to 2^-}\dfrac{x-2}{x-2}=\lim\limits_{x\to 2^-}1=1$.

Suy ra $\lim\limits_{x\to 2^-}\dfrac{|x-2|}{x-2}\ne\lim\limits_{x\to 2^+}\dfrac{|x-2|}{x-2}$ nên không tồn tại giới hạn của $B=\lim\limits_{x\to 2}\dfrac{|x-2|}{x-2}$.

3) $C=\lim\limits_{x\to 3}\dfrac{\left|x^2-9\right|}{x-3}$.

Ta có $C=\lim\limits_{x\to 3}\dfrac{|x-3|.|x+3|}{x-3}$. Do đó:

+) $\lim\limits_{x\to 3^+}\dfrac{\left|x^2-9\right|}{x-3}=\lim\limits_{x\to 3^+}\dfrac{(x-3).|x+3|}{x-3}=\lim\limits_{x\to 3^+}|x+3|=6.$

+) $\lim\limits_{x\to 3^-}\dfrac{\left|x^2-9\right|}{x-3}=\lim\limits_{x\to 3^-}\dfrac{-(x-3).|x+3|}{x-3}=\lim\limits_{x\to 3^-}\left(-|x+3|\right)=-6.$

Suy ra giới hạn của $C=\lim\limits_{x\to 3}\dfrac{\left|x^2-9\right|}{x-3}$ không tồn tại.

Bài 2. 1) $C=\lim\limits_{x\to 1^-}\dfrac{2x^2-2x+|x-1|\sqrt{x+3}}{x^2-2x+1}$.

Vì $x\to 1^-\Rightarrow x-1<0\Rightarrow |x-1|=-(x-1)$. Do đó

$$C=\lim\limits_{x\to 1^-}\frac{2x(x-1)-(x-1)\sqrt{x+3}}{(x-1)^2}=\lim\limits_{x\to 1^-}\frac{2x-\sqrt{x+3}}{x-1}=\lim\limits_{x\to 1^-}\frac{4x^2-x-3}{(x-1)\left(2x+\sqrt{x+3}\right)}$$

$$=\lim\limits_{x\to 1^-}\frac{(x-1)(4x+3)}{(x-1)\left(2x+\sqrt{x+3}\right)}=\lim\limits_{x\to 1^-}\frac{4x+3}{2x+\sqrt{x+3}}=\frac{7}{4}.$$

2) $C=\lim\limits_{x\to 2^-}\dfrac{|x-2|}{\sqrt{x-1}-1}$.

Vì $x\to 2^-\Rightarrow x-2<0\Rightarrow |x-2|=-(x-2)$. Do đó:

$$C=\lim\limits_{x\to 2^-}\frac{-(x-2)\left(\sqrt{x-1}+1\right)}{(x-1)-1}=\lim\limits_{x\to 2^-}\left[-\left(\sqrt{x-1}+1\right)\right]=-2.$$

3) $D=\lim\limits_{x\to 3^-}\dfrac{\sqrt{x^2-7x+12}}{\sqrt{9-x^2}}$.

Ta có $D=\lim\limits_{x\to 3^-}\dfrac{\sqrt{(x-3)(x-4)}}{\sqrt{(3-x)(3+x)}}=\lim\limits_{x\to 3^-}\dfrac{\sqrt{3-x}.\sqrt{4-x}}{\sqrt{3-x}.\sqrt{3+x}}=\lim\limits_{x\to 3^-}\dfrac{\sqrt{4-x}}{\sqrt{3+x}}=\dfrac{1}{\sqrt{6}}$.

4) $D=\lim\limits_{x\to 2^-}\dfrac{\sqrt{x^2-5x+6}}{\sqrt{4-x^2}}$.

Ta có $D=\lim\limits_{x\to 2^-}\dfrac{\sqrt{(x-2)(x-3)}}{\sqrt{(2-x)(2+x)}}=\lim\limits_{x\to 2^-}\dfrac{\sqrt{2-x}.\sqrt{3-x}}{\sqrt{2-x}.\sqrt{2+x}}=\lim\limits_{x\to 2^-}\dfrac{\sqrt{3-x}}{\sqrt{2+x}}=\dfrac{1}{2}$.

5) $D=\lim\limits_{x\to 1^-}\dfrac{\sqrt{1-x}+x-1}{\sqrt{x^2-x^3}}$.

Ta có $D=\lim\limits_{x\to 1^-}\dfrac{\sqrt{1-x}-(1-x)}{\sqrt{x^2(1-x)}}=\lim\limits_{x\to 1^-}\dfrac{\sqrt{1-x}-\sqrt{(1-x)^2}}{|x|\sqrt{1-x}}=\lim\limits_{x\to 1^-}\dfrac{1-\sqrt{1-x}}{|x|}=1$.

6) $D=\lim\limits_{x\to 1^+}(1-x)\sqrt{\dfrac{x+5}{x^3+2x^2-3}}$.

Ta có $D=\lim\limits_{x\to 1^+}\left[-\sqrt{\dfrac{(x-1)^2(x+5)}{(x-1)(x^2+3x+3)}}\right]=\lim\limits_{x\to 1^+}\left[-\sqrt{\dfrac{(x-1)(x+5)}{x^2+3x+3}}\right]=0$.

7) $D=\lim\limits_{x\to 1^-}\dfrac{\sqrt{x^3-3x+2}}{x^2-5x+4}$.

Ta có $D=\lim\limits_{x\to 1^-}\dfrac{\sqrt{(x-1)^2(x+2)}}{(x-1)(x-4)}=\lim\limits_{x\to 1^-}\dfrac{(1-x)\sqrt{x+2}}{(x-1)(x-4)}=\lim\limits_{x\to 1^-}\dfrac{\sqrt{x+2}}{4-x}=\dfrac{\sqrt{3}}{3}$.

Bài 3. 1) Ta có:

+) $\lim\limits_{x\to 1^-}f(x)=\lim\limits_{x\to 1^-}(x^3-3x)=-2$.

+) $\lim\limits_{x\to 1^+}f(x)=\lim\limits_{x\to 1^+}(5x^4-6x^2-x)=5-6-1=-2$.

+) Vì $\lim\limits_{x\to 1^-}f(x)=\lim\limits_{x\to 1^+}f(x)=-2$ nên hàm số $f(x)$ có giới hạn tại $x=1$ và $\lim\limits_{x\to 1}f(x)=-2$.

2) Ta có:

+) $\lim\limits_{x\to 1^-}f(x)=\lim\limits_{x\to 1^-}(x-3)=-2$.

+) $\lim\limits_{x\to 1^+}f(x)=\lim\limits_{x\to 1^+}\left(1-\sqrt{7x^2+2}\right)=-2$.

+) Vì $\lim\limits_{x\to 1^-}f(x)=\lim\limits_{x\to 1^+}f(x)=-2$ nên $C=\lim\limits_{x\to 1}f(x)=-2$.

3) Ta có:

+) $\lim\limits_{x\to(-2)^-}f(x)=\lim\limits_{x\to(-2)^-}\dfrac{3x-2}{x+1}=8$.

+) $\lim\limits_{x\to(-2)^+}f(x)=\lim\limits_{x\to(-2)^+}(x+10)=8$.

+)Vì $\lim\limits_{x\to(-2)^-}f(x)=\lim\limits_{x\to(-2)^+}f(x)=8$ nên $C=\lim\limits_{x\to -2}f(x)=8$.

Bài 4. Ta có:

+) $\lim\limits_{x\to(-1)^-} f(x)=\lim\limits_{x\to(-1)^-}\dfrac{x^3+1}{x+1}=\lim\limits_{x\to(-1)^-}\left(x^2-x+1\right)=3.$

+) $\lim\limits_{x\to(-1)^+} f(x)=\lim\limits_{x\to(-1)^+}\left(mx^2-x+m^2\right)=m^2+m+1.$

+) Để hàm số có giới hạn tại $x=-1$ thì

$$3=m^2+m+1\Leftrightarrow m^2+m-2=0\Leftrightarrow\left[\begin{array}{l}m=1\\m=-2\end{array}\right..$$

◆Dạng 5. Giới hạn của hàm số lượng giác

Phương pháp giải:

- *Sử dụng các định lý về giới hạn hàm số*
- *Sử dụng các công thức biến đổi lượng giác*
- *Lưu ý:* $\lim\limits_{x\to0}\dfrac{\sin x}{x}=1$

❶ VÍ DỤ

Ví dụ 1. Tính giới hạn $A=\lim\limits_{x\to\frac{\pi}{6}}\dfrac{2\sin x-1}{4\cos^2 x-3}$. **Đs:** $A=-\dfrac{1}{2}$.

Lời giải

Ta có: $A=\lim\limits_{x\to\frac{\pi}{6}}\dfrac{2\sin x-1}{4\cos^2 x-3}=\lim\limits_{x\to\frac{\pi}{6}}\dfrac{2\sin x-1}{4\left(1-\sin^2 x\right)-3}=\lim\limits_{x\to\frac{\pi}{6}}\dfrac{2\sin x-1}{1-4\sin^2 x}=\lim\limits_{x\to\frac{\pi}{6}}\dfrac{-1}{1+2\sin x}=-\dfrac{1}{2}.$

Ví dụ 2. Tính giới hạn $A=\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\sqrt{2}\sin x-1}{2\cos^2 x-1}$. **Đs:** $A=-\dfrac{1}{2}$.

Lời giải

Ta có: $A=\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\sqrt{2}\sin x-1}{2\cos^2 x-1}=\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\sqrt{2}\sin x-1}{2\left(1-\sin^2 x\right)-1}=\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\sqrt{2}\sin x-1}{1-2\sin^2 x}=\lim\limits_{x\to\frac{\pi}{4}}\dfrac{-1}{1+\sqrt{2}\sin x}=-\dfrac{1}{2}.$

Ví dụ 3. Tính giới hạn $A=\lim\limits_{x\to0}\dfrac{\cos 4x-1}{\sin 4x}$. **Đs:** $A=0$.

Lời giải

Ta có: $A=\lim\limits_{x\to0}\dfrac{\cos 4x-1}{\sin 4x}=\lim\limits_{x\to0}\dfrac{\cos^2 2x-\sin^2 2x-\cos^2 2x-\sin^2 2x}{2\sin 2x\cos 2x}$

$=\lim\limits_{x\to0}\dfrac{-2\sin^2 2x}{2\sin 2x\cos 2x}=\lim\limits_{x\to0}\dfrac{-\sin 2x}{\cos 2x}=0.$

Ví dụ 4. Tính giới hạn $A=\lim\limits_{x\to0}\dfrac{1-\sin 2x-\cos 2x}{1+\sin 2x-\cos 2x}$. **Đs:** $A=-1$.

Lời giải

Ta có: $A=\lim\limits_{x\to 0}\frac{1-\sin 2x-\cos 2x}{1+\sin 2x-\cos 2x}=\lim\limits_{x\to 0}\frac{1-2\sin x\cos x-\left(\cos^2 x-\sin^2 x\right)}{1+2\sin x\cos x-\left(\cos^2 x-\sin^2 x\right)}$.

$=\lim\limits_{x\to 0}\frac{2\sin^2 x-2\sin x\cos x}{2\sin^2 x+2\sin x\cos x}=\lim\limits_{x\to 0}\frac{2\sin x\left(\sin x-\cos x\right)}{2\sin x\left(\sin x+\cos x\right)}=\lim\limits_{x\to 0}\frac{\sin x-\cos x}{\sin x+\cos x}=-1.$

❷ BÀI TẬP ÁP DỤNG

Bài 1. Tính các giới hạn sau:

1) $A=\lim\limits_{x\to 0}\frac{1+\sin 2x-\cos 2x}{1-\sin 2x-\cos 2x}$. **Đs:** $A=-1$.

2) $A=\lim\limits_{x\to 0}\frac{\sin 2x}{1-\sin 2x-\cos 2x}$. **Đs:** $A=-1$.

3) $A=\lim\limits_{x\to 0}\frac{\sin 7x-\sin 5x}{\sin x}$. **Đs:** $A=2$.

4) $A=\lim\limits_{x\to 0}\frac{\sin 5x-\sin 3x}{\sin x}$. **Đs:** $A=2$.

5) $A=\lim\limits_{x\to 0}\frac{1-\cos x}{\sin x}$. **Đs:** $A=0$.

6) $A=\lim\limits_{x\to \frac{\pi}{3}}\frac{\cos 3x+2\cos 2x+2}{\sin 3x}$. **Đs:** $A=\frac{2\sqrt{3}}{3}$.

7) $A=\lim\limits_{x\to \frac{\pi}{2}}\frac{1+\sin 2x+\cos 2x}{\cos x}$. **Đs:** $A=2$.

Bài 2. Tính các giới hạn sau:

1) $B=\lim\limits_{x\to 0}\frac{1-\cos ax}{1-\cos bx}$. **Đs:** $B=\left(\frac{a}{b}\right)^2$.

2) $B=\lim\limits_{x\to 0}\frac{\sin 5x}{x}$. **Đs:** $B=5$.

3) $B=\lim\limits_{x\to 0}\frac{\sin 5x.\sin 3x.\sin x}{45x^3}$. **Đs:** $B=\frac{1}{3}$.

4) $B=\lim\limits_{x\to 0}\frac{1-\cos x}{x^2}$. **Đs:** $B=\frac{1}{2}$.

5) $B=\lim\limits_{x\to 0}\frac{1-\cos 5x}{1-\cos 3x}$. **Đs:** $B=\frac{25}{9}$.

6) $B=\lim\limits_{x\to 0}\frac{1-\cos ax}{x^2}$. **Đs:** $B=\frac{a^2}{2}$.

7) $B=\lim\limits_{x\to 0}\frac{1-\cos^2 2x}{x\sin x}$. **Đs:** $B=4$.

8) $B=\lim\limits_{x\to 0}\frac{\sin x-\tan x}{x^3}$. **Đs:** $B=-\frac{1}{2}$.

9) $B=\lim\limits_{x\to 0}\frac{\tan x-\sin x}{\sin^3 x}$. **Đs:** $B=\frac{1}{2}$.

10) $B=\lim\limits_{x\to 0}\frac{1-\cos^3 x}{x\sin x}$. **Đs:** $B=\frac{3}{2}$.

Bài 3. Tính các giới hạn sau:

1) $B=\lim\limits_{x\to 0}\frac{\left(\sqrt{\cos 8x}-1\right)\sin^2 3x}{3.x^4}$. **Đs:** $B=-48$.

2) $B=\lim\limits_{x\to 0}\frac{1-\sqrt{2x+1}}{\sin 2x}$. **Đs:** $B=\frac{-1}{2}$.

3) $B=\lim\limits_{x\to 0}\frac{1-\cos\sqrt{\cos 2x}}{x^2}$. **Đs:** $B=\frac{3}{2}$.

4) $B=\lim\limits_{x\to 0}\frac{1-\sqrt[3]{\cos x}}{\tan^2 x}$. **Đs:** $B=\frac{1}{6}$.

5) $B=\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\sqrt[3]{\tan x}-1}{2\sin^2 x-1}$. **Đs:** $B=\dfrac{1}{3}$.

6) $B=\lim\limits_{x\to 0}\dfrac{\sqrt{1+\tan x}-\sqrt{1+\sin x}}{x^3}$. **Đs:** $B=\dfrac{1}{4}$. 7) $B=\lim\limits_{x\to 0}\dfrac{1-\cos x}{\left(1-\sqrt{1-x}\right)^2}$. **Đs:** $B=2$.

8) $B=\lim\limits_{x\to 0}\dfrac{\sqrt{1+x^2}-\cos x}{x^2}$. **Đs:** $B=1$. 9) $B=\lim\limits_{x\to 0}\dfrac{1-\sqrt{2x+1}+\sin x}{\sqrt{3x+4}-2-x}$. **Đs:** $B=0$.

10) $B=\lim\limits_{x\to 0}\dfrac{\sqrt{2x+1}-\sqrt[3]{x^2+1}}{\sin x}$. **Đs:** 1.

Bài 4. Tính các giới hạn sau:

1) $C=\lim\limits_{x\to\frac{\pi}{4}}\left[\tan 2x\tan\left(\dfrac{\pi}{4}-x\right)\right]$. **Đs:** $C=\dfrac{1}{2}$

2) $C=\lim\limits_{x\to\pi}\dfrac{1+\cos x}{(x-\pi)^2}$. **Đs:** $C=\dfrac{1}{2}$

3) $C=\lim\limits_{x\to\pi}\dfrac{\sin(x-1)}{x^2-4x+3}$. **Đs:** $C=\dfrac{-1}{2}$.

4) $C=\lim\limits_{x\to a}\dfrac{\sin x-\sin a}{x-a}$. **Đs:** $C=\cos a$.

❸ LỜI GIẢI

Bài 1. 1) $A=\lim\limits_{x\to 0}\dfrac{1+\sin 2x-\cos 2x}{1-\sin 2x-\cos 2x}=\lim\limits_{x\to 0}\dfrac{1+2\sin x\cos x-(\cos^2 x-\sin^2 x)}{1-2\sin x\cos x-(\cos^2 x-\sin^2 x)}$.

$=\lim\limits_{x\to 0}\dfrac{2\sin^2 x+2\sin x\cos x}{2\sin^2 x-2\sin x\cos x}=\lim\limits_{x\to 0}\dfrac{2\sin x(\sin x+\cos x)}{2\sin x(\sin x-\cos x)}=\lim\limits_{x\to 0}\dfrac{\sin x+\cos x}{\sin x-\cos x}=-1$.

2) $A=\lim\limits_{x\to 0}\dfrac{\sin 2x}{1-\sin 2x-\cos 2x}=\lim\limits_{x\to 0}\dfrac{2\sin x\cos x}{1-2\sin x\cos x-(\cos^2 x-\sin^2 x)}$.

$=\lim\limits_{x\to 0}\dfrac{2\sin x\cos x}{2\sin^2 x-2\sin x\cos x}=\lim\limits_{x\to 0}\dfrac{2\sin x\cos x}{2\sin x(\sin x-\cos x)}=\lim\limits_{x\to 0}\dfrac{\cos x}{\sin x-\cos x}=-1$.

3) $A=\lim\limits_{x\to 0}\dfrac{\sin 7x-\sin 5x}{\sin x}=\lim\limits_{x\to 0}\dfrac{2\cos 6x.\sin x}{\sin x}=\lim\limits_{x\to 0}2\cos 6x=2$.

4) $A=\lim\limits_{x\to 0}\dfrac{\sin 5x-\sin 3x}{\sin x}=\lim\limits_{x\to 0}\dfrac{2\cos 4x.\sin x}{\sin x}=\lim\limits_{x\to 0}2\cos 4x=2$.

5) $A=\lim\limits_{x\to 0}\dfrac{1-\cos x}{\sin x}=\lim\limits_{x\to 0}\dfrac{2\sin^2\frac{x}{2}}{2\sin\frac{x}{2}.\cos\frac{x}{2}}=\lim\limits_{x\to 0}\dfrac{\sin\frac{x}{2}}{\cos\frac{x}{2}}=0.$

6) $A=\lim\limits_{x\to\frac{\pi}{3}}\dfrac{\cos 3x+2\cos 2x+2}{\sin 3x}=\lim\limits_{x\to\frac{\pi}{3}}\dfrac{4\cos^3 x-3\cos x+2\left(\cos^2 x-\sin^2 x\right)+2}{3\sin x-4\sin^3 x}$

$=\lim\limits_{x\to\frac{\pi}{3}}\dfrac{4\cos^3 x-3\cos x+4\cos^2 x}{\sin x\left(3-4\sin^2 x\right)}=\lim\limits_{x\to\frac{\pi}{3}}\dfrac{\cos x\left(4\cos^2 x-3+4\cos x\right)}{\sin x\left[3-4\left(1-\cos^2 x\right)\right]}$

$=\lim\limits_{x\to\frac{\pi}{3}}\dfrac{\cos x\left[\left(2\cos x+1\right)^2-4\right]}{\sin x\left[4\cos^2 x-1\right]}=\lim\limits_{x\to\frac{\pi}{3}}\dfrac{\cos x\left(2\cos x+3\right)\left(2\cos x-1\right)}{\sin x\left(2\cos x-1\right)\left(2\cos x+1\right)}=\lim\limits_{x\to\frac{\pi}{3}}\dfrac{\cos x\left(2\cos x+3\right)}{\sin x\left(2\cos x+1\right)}=\dfrac{2\sqrt{3}}{3}.$

7) $A=\lim\limits_{x\to\frac{\pi}{2}}\dfrac{1+\sin 2x+\cos 2x}{\cos x}=\lim\limits_{x\to\frac{\pi}{2}}\dfrac{2\cos^2 x+2\sin x\cos x}{\cos x}$

$=\lim\limits_{x\to\frac{\pi}{2}}\dfrac{2\cos x\left(\cos x+\sin x\right)}{\cos x}=\lim\limits_{x\to\frac{\pi}{2}}2\left(\cos x+\sin x\right)=2.$

Bài 2. 1) $A=\lim\limits_{x\to 0}\dfrac{1-\cos ax}{1-\cos bx}=\lim\limits_{x\to 0}\dfrac{2\sin^2\frac{ax}{2}}{2\sin^2\frac{bx}{2}}=\lim\limits_{x\to 0}\left(\dfrac{a}{b}.\dfrac{\sin\frac{ax}{2}}{\frac{ax}{2}}.\dfrac{\frac{bx}{2}}{\sin\frac{bx}{2}}\right)^2=\left(\dfrac{a}{b}\right)^2.$

(*Vì* $\lim\limits_{x\to 0}\dfrac{\sin\frac{ax}{2}}{\frac{ax}{2}}=1$ *và* $\lim\limits_{x\to 0}\dfrac{\frac{bx}{2}}{\sin\frac{bx}{2}}=1$).

2) $B=\lim\limits_{x\to 0}\dfrac{\sin 5x}{x}=\lim\limits_{x\to 0}\left(5.\dfrac{\sin 5x}{5x}\right)=5$. (*Vì* $\lim\limits_{x\to 0}\dfrac{\sin 5x}{5x}=1$).

3) $B=\lim\limits_{x\to 0}\dfrac{\sin 5x.\sin 3x.\sin x}{45x^3}=\lim\limits_{x\to 0}\left(\dfrac{\sin 5x}{5x}.\dfrac{\sin 3x}{3x}.\dfrac{\sin x}{x}.\dfrac{1}{3}\right)=\dfrac{1}{3}$

(*Vì* $\lim\limits_{x\to 0}\dfrac{\sin 5x}{5x}=1,\ \lim\limits_{x\to 0}\dfrac{\sin 3x}{3x}=1,\ \lim\limits_{x\to 0}\dfrac{\sin x}{x}=1$).

4) $B=\lim\limits_{x\to 0}\dfrac{1-\cos x}{x^2}=\lim\limits_{x\to 0}\dfrac{2\sin^2\frac{x}{2}}{\left(\frac{x}{2}\right)^2.4}=\dfrac{1}{2}$, (*vì* $\lim\limits_{x\to 0}\dfrac{\sin^2\frac{x}{2}}{\left(\frac{x}{2}\right)^2}=1$.

5) $B=\lim\limits_{x\to 0}\dfrac{1-\cos 5x}{1-\cos 3x}=\lim\limits_{x\to 0}\dfrac{2\sin^2\frac{5x}{2}}{2\sin^2\frac{3x}{2}}=\lim\limits_{x\to 0}\left[\dfrac{\sin^2\frac{5x}{2}.\left(\frac{3x}{2}\right)^2}{\left(\frac{5x}{2}\right)^2.\sin^2\frac{3x}{2}}.\dfrac{25}{9}\right]=\dfrac{25}{9}$

$$\left(Vì\ \lim_{x\to 0}\frac{\sin^2\frac{5x}{2}}{\left(\frac{5x}{2}\right)^2}=1\ và\ \lim_{x\to 0}\frac{\left(\frac{3x}{2}\right)^2}{\sin^2\frac{3x}{2}}=1\right).$$

6) $$B=\lim_{x\to 0}\frac{1-\cos ax}{x^2}=\lim_{x\to 0}\left[\frac{2\sin^2\frac{ax}{2}}{\left(\frac{ax}{2}\right)^2}.\frac{a^2}{4}\right]=\frac{a^2}{2},\ \left(vì\ \lim_{x\to 0}\frac{\sin^2\frac{ax}{2}}{\left(\frac{ax}{2}\right)^2}=1\right).$$

7) $$B=\lim_{x\to 0}\frac{\sin^2 2x}{x.\sin x}=\lim_{x\to 0}\frac{4\sin^2 x.\cos^2 x}{x.\sin x}=\lim_{x\to 0}\left(\frac{\sin x}{x}.4\cos^2 x\right)=4,\ \left(vì\ \lim_{x\to 0}\frac{\sin x}{x}=1\right).$$

8) $$B=\lim_{x\to 0}\frac{\sin x-\tan x}{x^3}=\lim_{x\to 0}\frac{\sin x-\frac{\sin x}{\cos x}}{x^3}=\lim_{x\to 0}\frac{\sin x.\cos x-\sin x}{x^3\cos x}$$

$$=\lim_{x\to 0}\frac{-\sin x(1-\cos x)}{x^3\cos x}=\lim_{x\to 0}\left[\frac{-2\sin x}{x}.\frac{\sin^2\frac{x}{2}}{\left(\frac{x}{2}\right)^2}.\frac{-2}{4\cos x}\right]=\frac{-1}{2}.$$

$$\left(vì\ \lim_{x\to 0}\frac{\sin x}{x}=1\ và\ \lim_{x\to 0}\frac{\sin^2\left(\frac{x}{2}\right)}{\left(\frac{x}{2}\right)^2}=1\right).$$

9) $$B=\lim_{x\to 0}\frac{\tan x-\sin x}{\sin^3 x}=\lim_{x\to 0}\frac{\frac{\sin x}{\cos x}-\sin x}{\sin^3 x}=\lim_{x\to 0}\frac{\sin x-\sin x.\cos x}{\sin^3 x\cos x}$$

$$=\lim_{x\to 0}\frac{1-\cos x}{\sin^2 x.\cos x}=\lim_{x\to 0}\frac{2\sin^2\frac{x}{2}}{4.\sin^2\frac{x}{2}.\cos^2\frac{x}{2}.\cos x}=\lim_{x\to 0}\frac{1}{\cos^2\frac{x}{2}.\cos x}=\frac{1}{2}$$

10) $$B=\lim_{x\to 0}\frac{(1-\cos x)\left(1+\cos x+\cos^2 x\right)}{x\sin x}=\lim_{x\to 0}\frac{2\sin^2\frac{x}{2}\left(1+\cos x+\cos^2 x\right)}{2x.\sin\frac{x}{2}.\cos\frac{x}{2}}$$

$$=\lim_{x\to 0}\left(\frac{\sin\frac{x}{2}}{\frac{x}{2}}.\frac{1+\cos x+\cos^2 x}{2\cos\frac{x}{2}}\right)=\frac{3}{2},\ \left(vì\ \lim_{x\to 0}\frac{\sin\frac{x}{2}}{\frac{x}{2}}=1\right).$$

Bài 3. 1) $$B=\lim_{x\to 0}\frac{\left(\sqrt{\cos 8x}-1\right)\sin^2 3x}{3x^4}=\lim_{x\to 0}\frac{(\cos 8x-1)\sin^2 3x}{3x^4\left(\sqrt{\cos 8x}+1\right)}=\lim_{x\to 0}\frac{-2\sin^2 4x\sin^2 3x}{3x^4\left(\sqrt{\cos 8x}+1\right)}$$

$$= \lim_{x\to 0}\left[\left(\frac{\sin 4x}{4x}\right)^2.\left(\frac{\sin 3x}{3x}\right)^2.\frac{-96}{\sqrt{\cos 8x+1}}\right]=-48$$

2) $$B=\lim_{x\to 0}\frac{1-\sqrt{2x+1}}{\sin 2x}=\lim_{x\to 0}\left[\frac{2x}{\sin 2x}.\frac{-1}{1+\sqrt{2x+1}}\right]=-\frac{1}{2}$$

3) $$B=\lim_{x\to 0}\frac{1-\cos x\sqrt{\cos 2x}}{x^2}=\lim_{x\to 0}\frac{1-\cos^2 x\cos 2x}{x^2\left(1+\cos x\sqrt{\cos 2x}\right)}=\lim_{x\to 0}\frac{1-\cos^2\left(1-2\sin^2 x\right)}{x^2\left(1+\cos x\sqrt{\cos 2x}\right)}$$

$$=\lim_{x\to 0}\frac{\sin^2 x+\cos^2 x-\cos^2 x\left(1-2\sin^2 x\right)}{x^2\left(1+\cos x\sqrt{\cos 2x}\right)}=\lim_{x\to 0}\frac{\sin^2 x+2\sin^2 x\cos^2 x}{x^2\left(1+\cos x\sqrt{\cos 2x}\right)}$$

$$=\lim_{x\to 0}\left[\left(\frac{\sin x}{x}\right)^2.\frac{1+2\cos^2 x}{1+\cos x\sqrt{\cos 2x}}\right]=\frac{3}{2}.$$

4) $$B=\lim_{x\to 0}\frac{1-\sqrt[3]{\cos x}}{\tan^2 x}=\lim_{x\to 0}\frac{1-\cos x}{\frac{\sin^2 x}{\cos^2 x}\left(1+\sqrt[3]{\cos x}+\sqrt[3]{\cos^2 x}\right)}$$

$$=\lim_{x\to 0}\frac{4\sin^2\frac{x}{2}\cos^2 x}{2\sin^2\frac{x}{x}\cos^2\frac{x}{2}\left(1+\sqrt[3]{\cos x}+\sqrt[3]{\cos^2 x}\right)}=\lim_{x\to 0}\frac{\cos^2 x}{2\cos^2\frac{x}{2}\left(1+\sqrt[3]{\cos x}+\sqrt[3]{\cos^2 x}\right)}=\frac{1}{6}.$$

5) $$B=\lim_{x\to\frac{\pi}{4}}\frac{\sqrt[3]{\tan x}-1}{2\sin^2 x-1}=\lim_{x\to\frac{\pi}{4}}\frac{\tan x-1}{\left(\sin^2 x-\cos^2 x\right)\left(\sqrt[3]{\tan^2 x}+\sqrt[3]{\tan x}+1\right)}$$

$$=\lim_{x\to\frac{\pi}{4}}\frac{\frac{\sin x-\cos x}{\cos x}}{\left(\sin^2 x-\cos^2 x\right)\left(\sqrt[3]{\tan^2 x}+\sqrt[3]{\tan x}+1\right)}=\lim_{x\to\frac{\pi}{4}}\frac{1}{\cos x\left(\sin x+\cos x\right)\left(\sqrt[3]{\tan^2 x}+\sqrt[3]{\tan x}+1\right)}=\frac{1}{3}.$$

6) $$B=\lim_{x\to 0}\frac{\sqrt{1+\tan x}-\sqrt{1+\sin x}}{x^3}=\lim_{x\to 0}\frac{\tan x-\sin x}{x^3\left(\sqrt{1+\tan x}+\sqrt{1+\sin x}\right)}$$

$$=\lim_{x\to 0}\frac{\sin x-\sin x\cos x}{x^3\cos x\left(\sqrt{1+\tan x}+\sqrt{1+\sin x}\right)}=\lim_{x\to 0}\frac{2\sin x\sin^2\frac{x}{2}}{x^3\cos x\left(\sqrt{1+\tan x}+\sqrt{1+\sin x}\right)}$$

$$=\lim_{x\to 0}\left[\frac{\sin x}{x}.\left(\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right)^2.\frac{2}{4\left(\sqrt{1+\tan x}+\sqrt{1+\sin x}\right)}\right]=\frac{1}{4}$$

7) $B=\lim\limits_{x\to 0}\dfrac{1-\cos x}{\left(1-\sqrt{1-x}\right)^2}=\lim\limits_{x\to 0}\dfrac{2\sin^2\dfrac{x}{2}\left(1+\sqrt{1-x}\right)^2}{x^2}=\lim\limits_{x\to 0}\left[\left(\dfrac{\sin\dfrac{x}{2}}{\dfrac{x}{2}}\right)^2.\dfrac{2\left(1+\sqrt{1-x}\right)^2}{4}\right]=2$.

8) $B=\lim\limits_{x\to 0}\dfrac{\sqrt{1+x^2}-\cos x}{x^2}=\lim\limits_{x\to 0}\dfrac{1+x^2-\cos^2 x}{x^2\left(\sqrt{1+x^2}+\cos x\right)}=\lim\limits_{x\to 0}\dfrac{x^2+\sin^2 x}{x^2\left(\sqrt{1+x^2}+\cos x\right)}$

$=\lim\limits_{x\to 0}\left(\dfrac{\sin^2 x}{x^2}.\dfrac{1}{\sqrt{1+x^2}+\cos x}+\dfrac{1}{\sqrt{1+x^2}+\cos x}\right)=\dfrac{1}{2}+\dfrac{1}{2}=1$.

9) $B=\lim\limits_{x\to 0}\dfrac{1-\sqrt{2x+1}+\sin x}{\sqrt{3x+4}-2-x}=\lim\limits_{x\to 0}\dfrac{1-\sqrt{2x+1}}{\sqrt{3x+4}-2-x}+\lim\limits_{x\to 0}\dfrac{\sin x}{\sqrt{3x+4}-2-x}$

$=\lim\limits_{x\to 0}\dfrac{-2x\left(\sqrt{3x+4}+2+x\right)}{\left(-x^2-x\right)\left(1+\sqrt{2x+1}\right)}+\lim\limits_{x\to 0}\dfrac{\sin x\left(\sqrt{3x+4}+2+x\right)}{-x(x+1)}$

$=\lim\limits_{x\to 0}\dfrac{-2\left(\sqrt{3x+4}+2+x\right)}{(-x-1)\left(1+\sqrt{2x+1}\right)}+\lim\limits_{x\to 0}\left(\dfrac{\sin x}{x}.\dfrac{\sqrt{3x+4}+2+x}{-x-1}\right)$

$=4-4=0$.

10) $B=\lim\limits_{x\to 0}\dfrac{\sqrt{2x+1}-\sqrt[3]{x^2+1}}{\sin x}=\lim\limits_{x\to 0}\dfrac{\sqrt{2x+1}-1+1-\sqrt[3]{x^2+1}}{\sin x}=\lim\limits_{x\to 0}\dfrac{\sqrt{2x+1}-1}{\sin x}+\lim\limits_{x\to 0}\dfrac{1-\sqrt[3]{x^2+1}}{\sin x}$

$=\lim\limits_{x\to 0}\dfrac{2x}{\sin x\left(\sqrt{2x+1}+1\right)}+\lim\limits_{x\to 0}\dfrac{-x^2}{\sin x\left(1+\sqrt[3]{x^2+1}+\sqrt[3]{\left(x^2+1\right)^2}\right)}=1$.

Bài 4. 1) $C=\lim\limits_{x\to\frac{\pi}{4}}\left[\tan 2x\tan\left(\dfrac{\pi}{4}-x\right)\right]$

Đặt $t=x-\dfrac{\pi}{4}$, vì $x\to\dfrac{\pi}{4}\Rightarrow t\to 0$. Khi đó:

$C=\lim\limits_{t\to 0}\left[\tan\left(2t+\dfrac{\pi}{2}\right)(-1)\tan t\right]=\lim\limits_{t\to 0}\left(\cot 2t\tan t\right)=\lim\limits_{t\to 0}\dfrac{\cos 2t}{2\cos^2 t}=\dfrac{1}{2}$.

2) $C=\lim\limits_{x\to\pi}\dfrac{1+\cos x}{(x-\pi)^2}$

Đặt $t=x-\pi$, vì $x\to\pi\Rightarrow t\to 0$. Khi đó:

$C=\lim\limits_{t\to 0}\dfrac{1-\cos t}{t^2}=\lim\limits_{t\to 0}\dfrac{2\sin^2\dfrac{t}{2}}{t^2}=\dfrac{1}{2}$.

3) $C=\lim\limits_{x\to\pi}\dfrac{\sin(x-1)}{x^2-4x+3}$

Đặt $t=x-\pi$, vì $x\to 1\Rightarrow t\to 0$. Khi đó:

$$C=\lim_{x\to\pi}\frac{\sin(x-1)}{x^2-4x+3}=\lim_{x\to\pi}\frac{\sin(x-1)}{(x-1)(x-3)}=\lim_{t\to 0}\frac{\text{sint}}{t(t-2)}=-\frac{1}{2}.$$

4) $C=\lim_{x\to a}\frac{\sin x-\sin a}{x-a}$

Đặt $t=x-a$. vì $x\to a\Rightarrow t\to 0$. Khi đó:

$$C=\lim_{t\to 0}\frac{\sin(t+a)-\sin a}{t}=\lim_{t\to 0}\frac{2\cos\frac{t+2a}{2}.\sin\frac{t}{2}}{2.\frac{t}{2}}=\cos a.$$

C. BÀI TẬP RÈN LUYỆN

Bài 1. Tính các giới hạn sau:

1. $\lim_{x\to 3}\frac{x-3}{x^2-x-6}$. ĐS: $\frac{1}{5}$
2. $\lim_{x\to 3}\frac{x^2+2x-15}{x-3}$. ĐS : 8
3. $\lim_{x\to -3}\frac{x+3}{x^2+2x-3}$ ĐS: $\frac{1}{4}$
4. $\lim_{x\to 2}\frac{x^2-3x+2}{x^2-4}$. ĐS: $\frac{1}{4}$.
5. $\lim_{x\to -2}\frac{x^2+3x+2}{4-x^2}$. ĐS: $\frac{-1}{4}$
6. $\lim_{x\to 3}\frac{x^2-7x+12}{x^2-9}$. ĐS: $-\frac{1}{6}$.
7. $\lim_{x\to 1}\frac{x^2-1}{x^2+3x-4}$. ĐS: $\frac{2}{5}$
8. $\lim_{x\to 2}\frac{x^2+x-6}{x^2-4}$. ĐS: $\frac{5}{4}$.
9. $\lim_{x\to 2}\frac{2x^2+3x-14}{x^2-4}$. ĐS: $\frac{11}{4}$.
10. $\lim_{x\to 3}\frac{x^2-9}{x^2-4x+3}$. ĐS: 3
11. $\lim_{x\to 2}\frac{3x^2-x-10}{4x^2+x-18}$. ĐS: $\frac{11}{7}$.
12. $\lim_{x\to 5}\frac{x^2-5x}{x^2-25}$. ĐS: $\frac{1}{2}$.
13. $\lim_{x\to 2}\frac{4-x^2}{2x^2-10x+12}$. ĐS: 2
14. $\lim_{x\to 2}\frac{4-x^2}{2x^2-x-6}$. ĐS: $\frac{-4}{7}$.
15. $\lim_{x\to 3}\frac{x^2-5x+6}{x^2-3x}$. ĐS: $\frac{1}{3}$.
16. $\lim_{x\to 5}\frac{x^2-9x+20}{x^2-5x}$. ĐS: $\frac{1}{5}$.
17. $\lim_{x\to 3}\frac{3x^2-10x+3}{x^2-5x+6}$. ĐS: 8
18. $\lim_{x\to 3}\frac{x^2+2x-3}{2x^2-x-1}$. ĐS: 4
19. $\lim_{x\to 3}\frac{x^3-5x^2+6x}{9-x^2}$. ĐS: $-\frac{1}{2}$.
20. $\lim_{x\to -2}\frac{x^4-16}{x^2+6x+8}$. ĐS: -16.
21. $\lim_{x\to 2}\frac{8-x^3}{x^2-5x+6}$. ĐS: 12
22. $\lim_{x\to -2}\frac{8+x^3}{x^2+11x+18}$. ĐS: $\frac{12}{7}$.

23. $\lim\limits_{x\to\sqrt{2}}\dfrac{x^2-x-2+\sqrt{2}}{x^3-2\sqrt{2}}$. ĐS: $\dfrac{2\sqrt{2}-1}{6}$.

24. $\lim\limits_{x\to 2}\dfrac{x^3-8}{x^2-3x+2}$. ĐS: 12.

25. $\lim\limits_{x\to-\sqrt{2}}\dfrac{x^3+2\sqrt{2}}{x^2-2}$. ĐS: $-\dfrac{3\sqrt{2}}{2}$.

26. $\lim\limits_{x\to 0}\dfrac{(x+1)^3-1}{x}$. ĐS: 3.

27. $\lim\limits_{x\to 0}\dfrac{(x+1)^3-27}{x}$. ĐS: 27.

28. $\lim\limits_{x\to 3}\dfrac{x^4-27x}{2x^2-3x-9}$. ĐS: 9.

29. $\lim\limits_{x\to 2}\dfrac{x^3-5x^2+10x-8}{x-2}$. ĐS: 2.

30. $\lim\limits_{x\to 1}\dfrac{2x^3-5x^2+2x+1}{x^2-1}$. ĐS: −1.

31. $\lim\limits_{x\to 2}\dfrac{x^3-2x-4}{x^2-4}$. ĐS: $\dfrac{5}{2}$.

32. $\lim\limits_{x\to -2}\dfrac{x^3+3x^2+2x}{x^2-x-6}$. ĐS: $-\dfrac{2}{5}$.

33. $\lim\limits_{x\to -2}\dfrac{2x^2-x-10}{x^3-x+6}$. ĐS: $-\dfrac{9}{11}$.

34. $\lim\limits_{x\to 1}\dfrac{x^3-x^2-x+1}{x^2-2x+1}$. ĐS: 2.

35. $\lim\limits_{x\to 2}\dfrac{x^2-4}{x^3-3x-2}$. ĐS: $\dfrac{4}{9}$.

36. $\lim\limits_{x\to 2}\dfrac{x^3-2x^2-x+2}{x^2-4}$. ĐS: $\dfrac{3}{4}$.

37. $\lim\limits_{x\to 1}\dfrac{x^2+3x-4}{2x^3+x^2-3}$. ĐS: $\dfrac{5}{8}$

38. $\lim\limits_{x\to 1}\dfrac{3x^3-4x^2-2x+3}{3x^2-2x-1}$. ĐS: $-\dfrac{1}{4}$

39. $\lim\limits_{x\to 2}\dfrac{x^3+x^2-5x-2}{x^2-3x+2}$ ĐS: 11

40. $\lim\limits_{x\to 1}\dfrac{2x^3-5x+3}{x^2-3x+2}$ ĐS: -1

41. $\lim\limits_{x\to -2}\dfrac{x^2-2x-8}{3x^3+4x^2-x+6}$ ĐS: $-\dfrac{6}{19}$.

42. $\lim\limits_{x\to 1}\dfrac{1-x^3}{x^4-4x^2+3}$ ĐS: $\dfrac{3}{4}$.

43. $\lim\limits_{x\to 3}\dfrac{x^3-5x^2+3x+9}{x^4-8x^2-9}$ ĐS: 0.

44. $\lim\limits_{x\to \frac{1}{3}}\dfrac{6x^3-5x^2+4x-1}{9x^4+8x^2-1}$ ĐS: $\dfrac{2}{5}$.

45. $\lim\limits_{x\to 1}\dfrac{x+2\sqrt{x}-3}{x-5\sqrt{x}+4}$ ĐS: $-\dfrac{4}{3}$.

46. $\lim\limits_{x\to 1}\dfrac{x^3-3x+2}{x^4-4x+3}$ ĐS: $\dfrac{1}{2}$.

47. $\lim\limits_{x\to 2}\dfrac{x^5-2x^4+x-2}{x^2-4}$ ĐS: $\dfrac{17}{4}$.

48. $\lim\limits_{x\to 1}\dfrac{x^4-x^3-x+1}{x^3-5x^2+7x-3}$ ĐS: $-\dfrac{3}{2}$.

49. $\lim\limits_{x\to 3}\dfrac{2x^3-5x^2-2x-3}{4x^3-13x^2+4x-3}$ ĐS: $\dfrac{11}{17}$.

50. $\lim\limits_{x\to -1}\dfrac{2x^3+5x^2+4x+1}{x^3+x^2-x-1}$ ĐS: $\dfrac{1}{2}$.

51. $\lim\limits_{x\to 3}\dfrac{2x^3-5x^2-2x-3}{4x^3-12x^2+4x-12}$ ĐS: $\dfrac{11}{20}$.

52. $\lim\limits_{x\to 1}\dfrac{\sqrt[3]{x^2}-2\sqrt[3]{x}+1}{(x-1)^2}$ ĐS: $\dfrac{1}{9}$.

53. $\lim\limits_{x\to -2}\dfrac{2x^4+8x^3+7x^2-4x-4}{3x^3+14x^2+20x+8}$ ĐS: $-\dfrac{7}{4}$.

54. $\lim\limits_{x\to -\sqrt{3}}\dfrac{2x^3-3x^2+x+9+7\sqrt{3}}{3-x^2}$ ĐS: $\dfrac{7\sqrt{3}}{6}$

55. $\lim\limits_{x\to 1}\dfrac{x^4-5x^3+9x^2-7x+2}{x^4-3x^3+x^2+3x-2}$ ĐS: 0.

56. $\lim\limits_{x\to 1}\dfrac{x^5+x^4+x^3+x^2+x-5}{x^2-1}$ ĐS: $\dfrac{15}{2}$.

57. $\lim_{x\to 1}\left(\frac{1}{x-1}-\frac{2}{x^2-1}\right)$ ĐS: $\frac{1}{2}$.

58. $\lim_{x\to 2}\left(\frac{1}{x-2}-\frac{12}{x^3-8}\right)$ ĐS: $\frac{1}{2}$.

59. $\lim_{x\to 2}\left(\frac{1}{x^2-3x+2}+\frac{1}{x^2-5x+6}\right)$ ĐS: -2.

60. $\lim_{x\to -2}\left(\frac{2x-3}{x+2}-\frac{x-26}{4-x^2}\right)$ ĐS: $\frac{7}{2}$.

61. $\lim_{x\to 1}\left(\frac{1}{x^2+x-2}-\frac{1}{x^3-1}\right)$ ĐS: $\frac{2}{9}$.

62. $\lim_{x\to 0}\frac{(1+x)(1+2x)(1+3x)-1}{x}$ ĐS: 6.

63. $\lim_{x\to 1}\frac{x^n-1}{x^m-1}$ ĐS: $\frac{n}{m}$.

64. $\lim_{x\to 1}\frac{x^n-nx+n-1}{(x-1)^2}$ ĐS: $\frac{(n-2)(n-1)}{2}$.

65. $\lim_{x\to 1}\frac{x^{100}-2x+1}{x^{50}-2x+1}$ ĐS: 2.

66. $\lim_{x\to 1}\frac{x+x^2+...+x^n-n}{x-1}$ ĐS: $\frac{n(n+1)}{2}$.

Lời giải

1. $\lim_{x\to 3}\frac{x-3}{x^2-x-6}=\lim_{x\to 3}\frac{x-3}{(x+2)(x-3)}=\lim_{x\to 3}\frac{1}{x+2}=\frac{1}{5}.$

2. $\lim_{x\to 3}\frac{x^2+2x-15}{x-3}=\lim_{x\to 3}\frac{(x-3)(x+5)}{x-3}=\lim_{x\to 3}(x+5)=8$

3. $\lim_{x\to -3}\frac{x+3}{x^2+2x-3}=\lim_{x\to -3}\frac{x+3}{(x+3)(x-1)}$

4. $\lim_{x\to 2}\frac{x^2-3x+2}{x^2-4}=\lim_{x\to 2}\frac{(x-1)(x-2)}{(x-2)(x+2)}=\lim_{x\to -3}\frac{1}{x-1}=\frac{1}{4}.=\lim_{x\to 2}\frac{x-1}{x+2}=\frac{1}{4}.$

5. $\lim_{x\to -2}\frac{x^2+3x+2}{4-x^2}=\lim_{x\to -2}\frac{(x+1)(x+2)}{(2-x)(2+x)}=\lim_{x\to -2}\frac{x+1}{2-x}=-\frac{1}{4}.$

6. $\lim_{x\to 3}\frac{x^2-7x+12}{x^2-9}=\lim_{x\to 3}\frac{(x-3)(x-4)}{(x-3)(x+3)}=\lim_{x\to 3}\frac{x-4}{x+3}=-\frac{1}{6}.$

7. $\lim_{x\to 1}\frac{x^2-1}{x^2+3x-4}=\lim_{x\to 1}\frac{(x-1)(x+1)}{(x-1)(x+4)}=\lim_{x\to 1}\frac{x+1}{x+4}=\frac{2}{5}$

8. $\lim_{x\to 2}\frac{x^2+x-6}{x^2-4}=\lim_{x\to 2}\frac{(x-2)(x+3)}{(x-2)(x+2)}=\lim_{x\to 2}\frac{x+3}{x+2}=\frac{5}{4}.$

9. $\lim_{x\to 2}\frac{2x^2+3x-14}{x^2-4}=\lim_{x\to 2}\frac{(x-2)(2x+7)}{(x-2)(x+2)}=\lim_{x\to 2}\frac{2x+7}{x+2}=\frac{11}{4}.$

10. $\lim_{x\to 3}\frac{x^2-9}{x^2-4x+3}=\lim_{x\to 3}\frac{(x-3)(x+3)}{(x-3)(x-1)}=\lim_{x\to 3}\frac{x+3}{x-1}=3.$

11. $\lim_{x\to 2}\frac{3x^2-x-10}{4x^2+x-18}=\lim_{x\to 2}\frac{(x-2)(3x+5)}{(x-2)(4x+9)}=\lim_{x\to 2}\frac{3x+5}{4x+9}=\frac{11}{17}.$

12. $\lim\limits_{x\to 5}\dfrac{x^2-5x}{x^2-25}=\lim\limits_{x\to 5}\dfrac{x(x-5)}{(x-5)(x+5)}=\lim\limits_{x\to 5}\dfrac{x}{x+5}=\dfrac{1}{2}.$

13. $\lim\limits_{x\to 2}\dfrac{4-x^2}{2x^2-10x+12}=\lim\limits_{x\to 2}\dfrac{(2-x)(2+x)}{2(x-2)(x-3)}=\lim\limits_{x\to 2}\dfrac{-x-2}{2(x-3)}=2.$

14. $\lim\limits_{x\to 2}\dfrac{4-x^2}{2x^2-x-6}=\lim\limits_{x\to 2}\dfrac{(2-x)(2+x)}{(x-2)(2x+3)}=\lim\limits_{x\to 2}\dfrac{-x-2}{2x+3}=\dfrac{-4}{7}.$

15. $\lim\limits_{x\to 3}\dfrac{x^2-5x+6}{x^2-3x}=\lim\limits_{x\to 3}\dfrac{(x-2)(x-3)}{x(x-2)}=\lim\limits_{x\to 3}\dfrac{x-2}{2}=\dfrac{1}{3}.$

16. $\lim\limits_{x\to 5}\dfrac{x^2-9x+20}{x^2-5x}=\lim\limits_{x\to 5}\dfrac{(x-4)(x-5)}{x(x-5)}=\lim\limits_{x\to 5}\dfrac{x-4}{x}=\dfrac{1}{5}.$

17. $\lim\limits_{x\to 3}\dfrac{x^2-5x+6}{x^2-3x}=\lim\limits_{x\to 3}\dfrac{(x-3)(3x-1)}{(x-2)(x-3)}=\lim\limits_{x\to 3}\dfrac{3x-1}{x-2}=8.$

18. $\lim\limits_{x\to 3}\dfrac{x^2+2x-3}{2x^2-x-1}=\lim\limits_{x\to 3}\dfrac{(x-1)(x+3)}{(x-1)(2x-1)}=\lim\limits_{x\to 3}\dfrac{x+3}{2x-1}=4.$

19. $\lim\limits_{x\to 3}\dfrac{x^3-5x^2+6x}{9-x^2}=\lim\limits_{x\to 3}\dfrac{x(x-2)(x-3)}{(3-x)(3+x)}=\lim\limits_{x\to 3}\dfrac{x(x-2)}{-x-3}=-\dfrac{1}{2}$

20. $\lim\limits_{x\to -2}\dfrac{x^4-16}{x^2+6x+8}=\lim\limits_{x\to -2}\dfrac{(x^2+4)(x-2)(x+2)}{(x+2)(4+x)}=\lim\limits_{x\to -2}\dfrac{(x^2+4)(x-2)}{x+4}=-16$

21. $\lim\limits_{x\to 2}\dfrac{8-x^3}{x^2-5x+6}=\lim\limits_{x\to 2}\dfrac{(2-x)(4+2x+x^2)}{(x-2)(x-3)}$

22. $\lim\limits_{x\to -2}\dfrac{8+x^3}{x^2+11x+18}=\lim\limits_{x\to -2}\dfrac{(x+2)(x^2-2x+4)}{(x+2)(x+9)}=\lim\limits_{x\to 2}\dfrac{-(x^2+2x+4)}{x-3}=12.$

$=\lim\limits_{x\to -2}\dfrac{x^2-2x+4}{x+9}=\dfrac{12}{7}.$

23. $\lim\limits_{x\to\sqrt{2}}\dfrac{x^2-x-2+\sqrt{2}}{x^3-2\sqrt{2}}=\lim\limits_{x\to\sqrt{2}}\dfrac{(x-\sqrt{2})(x-1+\sqrt{2})}{(x-\sqrt{2})(x^2+\sqrt{2}x+2)}=\lim\limits_{x\to\sqrt{2}}\dfrac{x-1+\sqrt{2}}{x^2+\sqrt{2}x+2}=\dfrac{2\sqrt{2}-1}{6}.$

24. $\lim\limits_{x\to 2}\dfrac{x^3-8}{x^2-3x+2}=\lim\limits_{x\to 2}\dfrac{(x-2)(x^2+2x+4)}{(x-1)(x-2)}=\lim\limits_{x\to 2}\dfrac{x^2+2x+4}{x-1}=12.$

25. $\lim\limits_{x\to -\sqrt{2}}\dfrac{x^3+2\sqrt{2}}{x^2-2}=\lim\limits_{x\to -\sqrt{2}}\dfrac{(x-\sqrt{2})(x^2-\sqrt{2}x+2)}{(x-\sqrt{2})(x+\sqrt{2})}=\lim\limits_{x\to -\sqrt{2}}\dfrac{x^2-\sqrt{2}x+2}{x-\sqrt{2}}=-\dfrac{3\sqrt{2}}{2}.$

26. $\lim\limits_{x\to 0}\dfrac{(x+1)^3-1}{x}=\lim\limits_{x\to 0}\dfrac{x^3+3x^2+3x}{x}=\lim\limits_{x\to 0}\left(x^2+3x+3\right)=3.$

27. $\lim\limits_{x\to 0}\dfrac{(x+1)^3-27}{x}=\lim\limits_{x\to 0}\dfrac{x\left[(x+3)^2+3(x+3)+9\right]}{x}=\lim\limits_{x\to 0}\left[(x+3)^2+3(x+3)+9\right]=27.$

28. $\lim\limits_{x\to 3}\dfrac{x^4-27x}{2x^2-3x-9}=\lim\limits_{x\to 3}\dfrac{x(x-3)\left(x^2+3x+9\right)}{(x-3)(2x+3)}=\lim\limits_{x\to 3}\dfrac{x\left(x^2+3x+9\right)}{2x+3}=9.$

29. $\lim\limits_{x\to 2}\dfrac{x^3-5x^2+10x-8}{x-2}=\lim\limits_{x\to 2}\dfrac{(x-2)\left(x^2-3x+4\right)}{x-2}=\lim\limits_{x\to 2}\left(x^2-3x+4\right)=2.$

30 $\lim\limits_{x\to 1}\dfrac{2x^3-5x^2+2x+1}{x^2-1}=\lim\limits_{x\to 1}\dfrac{(x-1)\left(2x^2-3x-1\right)}{(x-1)(x+1)}=\lim\limits_{x\to 1}\dfrac{2x^2-3x-1}{x+1}=-1.$

31. $\lim\limits_{x\to 2}\dfrac{x^3-2x-4}{x^2-4}=\lim\limits_{x\to 2}\dfrac{(x-2)\left(x^2+2x+2\right)}{(x-2)(x+2)}=\lim\limits_{x\to 2}\dfrac{x^2+2x+2}{x+2}=\dfrac{5}{2}.$

32. $\lim\limits_{x\to -2}\dfrac{x^3+3x^2+2x}{x^2-x-6}=\lim\limits_{x\to -2}\dfrac{x(x+1)(x+2)}{(x+2)(x-3)}=\lim\limits_{x\to -2}\dfrac{x(x+1)}{x-3}=-\dfrac{2}{5}.$

33. $\lim\limits_{x\to -2}\dfrac{2x^2-x-10}{x^3-x+6}=\lim\limits_{x\to -2}\dfrac{(x+2)(2x-5)}{(x+2)\left(x^2-2x+3\right)}=\lim\limits_{x\to -2}\dfrac{2x+5}{x^2-2x+3}=-\dfrac{9}{11}.$

34. $\lim\limits_{x\to 1}\dfrac{x^3-x^2-x+1}{x^2-2x+1}.=\lim\limits_{x\to 1}\dfrac{(x-1)^2(x+1)}{(x-1)^2}=\lim\limits_{x\to 1}(x+1)=2.$

35. $\lim\limits_{x\to 2}\dfrac{x^2-4}{x^3-3x-2}=\lim\limits_{x\to 2}\dfrac{(x-2)(x+2)}{(x-2)(x+1)^2}=\lim\limits_{x\to 2}\dfrac{x+2}{(x+1)^2}=\dfrac{4}{9}.$

36. $\lim\limits_{x\to 2}\dfrac{x^3-2x^2-x+2}{x^2-4}=\lim\limits_{x\to 2}\dfrac{(x-2)\left(x^2-1\right)}{(x-2)(x+2)}=\lim\limits_{x\to 2}\dfrac{x^2-1}{x+2}=\dfrac{3}{4}.$

37. $\lim\limits_{x\to 1}\dfrac{x^2+3x-4}{2x^3+x^2-3}=\lim\limits_{x\to 1}\dfrac{(x-1)(x+4)}{(x-1)(2x^2+3x+3)}=\lim\limits_{x\to 1}\dfrac{x+4}{2x^2+3x+3}=\dfrac{5}{8}$

38 $\lim\limits_{x\to 1}\dfrac{3x^3-4x^2-2x+3}{3x^2-2x-1}=\lim\limits_{x\to 1}\dfrac{(x-1)(3x^2-x-3)}{(x-1)(3x+1)}=\lim\limits_{x\to 1}\dfrac{3x^2-x-3}{3x+1}=-\dfrac{1}{4}$

39. $\lim\limits_{x\to 2}\dfrac{x^3+x^2-5x-2}{x^2-3x+2}=\lim\limits_{x\to 2}\dfrac{(x-2)(x^2+3x+1)}{(x-2)(x-1)}=\lim\limits_{x\to 2}\dfrac{x^2+3x+1}{x-1}=11.$

40. $\lim\limits_{x\to 1}\dfrac{2x^3-5x+3}{x^2-3x+2}=\lim\limits_{x\to 1}\dfrac{(x-1)(2x^2+2x-3)}{(x-2)(x-1)}=\lim\limits_{x\to 1}\dfrac{2x^2+2x-3}{x-2}=-1.$

41. $\lim\limits_{x\to -2}\dfrac{x^2-2x-8}{3x^3+4x^2-x+6}=\lim\limits_{x\to -2}\dfrac{(x+2)(x-4)}{(x+2)(3x^2-2x+3)}=\lim\limits_{x\to -2}\dfrac{x-4}{3x^2-2x+3}=-\dfrac{6}{19}.$

42. $\lim\limits_{x\to 1}\dfrac{1-x^3}{x^4-4x^2+3}=\lim\limits_{x\to 1}\dfrac{(x-1)(-x^2-x-1)}{(x-1)(x^3+x^2-3x-3)}=\lim\limits_{x\to 1}\dfrac{-x^2-x-1}{x^3+x^2-3x-3}=\dfrac{3}{4}$.

43. $\lim\limits_{x\to 3}\dfrac{x^3-5x^2+3x+9}{x^4-8x^2-9}=\lim\limits_{x\to 3}\dfrac{(x-3)(x^2-2x-3)}{(x-3)(x^3+3x^2+x+3)}=\lim\limits_{x\to 3}\dfrac{x^2-2x-3}{x^3+3x^2+x+3}=0$.

44. $\lim\limits_{x\to \frac{1}{3}}\dfrac{6x^3-5x^2+4x-1}{9x^4+8x^2-1}=\lim\limits_{x\to \frac{1}{3}}\dfrac{(3x-1)(2x^2-x+1)}{(3x-1)(3x^3+x^2+3x+1)}=\lim\limits_{x\to \frac{1}{3}}\dfrac{2x^2-x+1}{3x^3+x^2+3x+1}=\dfrac{2}{5}$.

45. $\lim\limits_{x\to 1}\dfrac{x+2\sqrt{x}-3}{x-5\sqrt{x}+4}=\lim\limits_{x\to 1}\dfrac{(\sqrt{x}-1)(\sqrt{x}+3)}{(\sqrt{x}-1)(\sqrt{x}-4)}=\lim\limits_{x\to 1}\dfrac{\sqrt{x}+3}{\sqrt{x}-4}=-\dfrac{4}{3}$.

46. $\lim\limits_{x\to 1}\dfrac{x^3-3x+2}{x^4-4x+3}=\lim\limits_{x\to 1}\dfrac{(x+2)(x^2-2x+1)}{(x^2-2x+1)(x^2+2x+3)}=\lim\limits_{x\to 1}\dfrac{x+2}{x^2+2x+3}=\dfrac{1}{2}$.

47. $\lim\limits_{x\to 2}\dfrac{x^5-2x^4+x-2}{x^2-4}=\lim\limits_{x\to 2}\dfrac{(x-2)(x^4+1)}{(x-2)(x+2)}=\lim\limits_{x\to 2}\dfrac{x^4+1}{x+2}=\dfrac{17}{4}$.

48. $\lim\limits_{x\to 1}\dfrac{x^4-x^3-x+1}{x^3-5x^2+7x-3}=\lim\limits_{x\to 1}\dfrac{(x^2-2x+1)(x^2+x+1)}{(x^2-2x+1)(x-3)}=\lim\limits_{x\to 1}\dfrac{x^2+x+1}{x-3}=-\dfrac{3}{2}$.

49. $\lim\limits_{x\to 3}\dfrac{2x^3-5x^2-2x-3}{4x^3-13x^2+4x-3}=\lim\limits_{x\to 3}\dfrac{(x-3)(2x^2+x+1)}{(x-3)(4x^2-x+1)}=\lim\limits_{x\to 3}\dfrac{2x^2+x+1}{4x^2-x+1}=\dfrac{11}{17}$.

50. $\lim\limits_{x\to -1}\dfrac{2x^3+5x^2+4x+1}{x^3+x^2-x-1}=\lim\limits_{x\to -1}\dfrac{(2x+1)(x^2+2x+1)}{(x-1)(x^2+2x+1)}=\lim\limits_{x\to -1}\dfrac{2x+1}{x-1}=\dfrac{1}{2}$.

51. $\lim\limits_{x\to 3}\dfrac{2x^3-5x^2-2x-3}{4x^3-12x^2+4x-12}=\lim\limits_{x\to 3}\dfrac{(x-3)(2x^2+x+1)}{4(x-3)(x^2+1)}=\lim\limits_{x\to 3}\dfrac{2x^2+x+1}{4(x^2+1)}=\dfrac{11}{20}$.

52. $\lim\limits_{x\to 1}\dfrac{\sqrt[3]{x^2}-2\sqrt[3]{x}+1}{(x-1)^2}=\lim\limits_{x\to 1}\dfrac{(\sqrt[3]{x}-1)^2}{(\sqrt[3]{x}-1)^2(\sqrt[3]{x^2}+\sqrt[3]{x}+1)^2}=\lim\limits_{x\to 1}\dfrac{1}{(\sqrt[3]{x^2}+\sqrt[3]{x}+1)^2}=\dfrac{1}{9}$.

53. $\lim\limits_{x\to -2}\dfrac{2x^4+8x^3+7x^2-4x-4}{3x^3+14x^2+20x+8}=\lim\limits_{x\to -2}\dfrac{(2x^2-1)(x^2+4x+4)}{(3x+2)(x^2+4x+4)}=\lim\limits_{x\to -2}\dfrac{2x^2-1}{3x+2}=-\dfrac{7}{4}$.

54. $\lim\limits_{x\to -\sqrt{3}}\dfrac{2x^3-3x^2+x+9+7\sqrt{3}}{3-x^2}=\lim\limits_{x\to -\sqrt{3}}\dfrac{(x+\sqrt{3})(2x^2-(3-2\sqrt{3})x+7-3\sqrt{3})}{(\sqrt{3}-x)(\sqrt{3}+x)}$

$=\lim\limits_{x\to -\sqrt{3}}\dfrac{2x^2-(3-2\sqrt{3})x+7-3\sqrt{3}}{\sqrt{3}-x}=\dfrac{7\sqrt{3}}{6}$

55. $\lim\limits_{x\to 1}\dfrac{x^4-5x^3+9x^2-7x+2}{x^4-3x^3+x^2+3x-2}=\lim\limits_{x\to 1}\dfrac{(x-1)^3(x-2)}{(x-1)^2(x-2)(x+1)}=\lim\limits_{x\to 1}\dfrac{x-1}{x+1}=0$.

56. $\lim\limits_{x\to 1}\dfrac{x^5+x^4+x^3+x^2+x-5}{x^2-1}=\lim\limits_{x\to 1}\dfrac{(x-1)(x^4+2x^3+3x^2+4x+5)}{(x-1)(x+1)}$

$=\lim\limits_{x\to 1}\dfrac{x^4+2x^3+3x^2+4x+5}{x+1}=\dfrac{15}{2}.$

57. $\lim\limits_{x\to 1}\left(\dfrac{1}{x-1}-\dfrac{2}{x^2-1}\right)=\lim\limits_{x\to 1}\dfrac{x-1}{(x-1)(x+1)}=\lim\limits_{x\to 1}\dfrac{1}{x+1}=\dfrac{1}{2}.$

58. $\lim\limits_{x\to 2}\left(\dfrac{1}{x-2}-\dfrac{12}{x^3-8}\right)=\lim\limits_{x\to 2}\dfrac{(x-2)(x+4)}{(x-2)(x^2+2x+4)}=\lim\limits_{x\to 2}\dfrac{x+4}{x^2+2x+4}=\dfrac{1}{2}.$

59. $\lim\limits_{x\to 2}\left(\dfrac{1}{x^2-3x+2}+\dfrac{1}{x^2-5x+6}\right)=\lim\limits_{x\to 2}\dfrac{2(x-2)}{(x-2)(x-3)(x-1)}=\lim\limits_{x\to 2}\dfrac{2}{(x-3)(x-1)}=-2.$

60. $\lim\limits_{x\to -2}\left(\dfrac{2x-3}{x+2}-\dfrac{x-26}{4-x^2}\right)=\lim\limits_{x\to -2}\dfrac{2(x-5)(x+2)}{(x-2)(x+2)}=\lim\limits_{x\to -2}\dfrac{2(x-5)}{x-2}=\dfrac{7}{2}.$

61. $\lim\limits_{x\to 1}\left(\dfrac{1}{x^2+x-2}-\dfrac{1}{x^3-1}\right)=\lim\limits_{x\to 1}\dfrac{(x-1)(x+1)}{(x-1)(x+2)(x^2+x+1)}=\lim\limits_{x\to 1}\dfrac{x+1}{(x+2)(x^2+x+1)}=\dfrac{2}{9}.$

62. $\lim\limits_{x\to 0}\dfrac{(1+x)(1+2x)(1+3x)-1}{x}=\lim\limits_{x\to 0}\dfrac{x(6x^2+11x+6)}{x}=\lim\limits_{x\to 0}\left(6x^2+11x+6\right)=6.$

63. $\lim\limits_{x\to 1}\dfrac{x^n-1}{x^m-1}=\lim\limits_{x\to 1}\dfrac{(x-1)(x^{n-1}+x^{n-2}+...+x+1)}{(x-1)(x^{m-1}+x^{m-2}+...+x+1)}=\lim\limits_{x\to 1}\dfrac{x^{n-1}+x^{n-2}+...+x+1}{x^{m-1}+x^{m-2}+...+x+1}=\dfrac{n}{m}.$

64. $\lim\limits_{x\to 1}\dfrac{x^n-nx+n-1}{(x-1)^2}=\lim\limits_{x\to 1}\dfrac{(x-1)(x^{n-1}+x^{n-2}+...+x+1)-\mathrm{n}(x-1)}{(x-1)^2}$

$=\lim\limits_{x\to 1}\dfrac{(x^{n-1}-1)+(x^{n-2}-1)+...+(x^2-1)+(x-1)}{x-1}$

$=\lim\limits_{x\to 1}\left((x^{n-2}+x^{n-3}+...+x+1)+(x^{n-3}+x^{n-4}+...+x+1)+...+1\right)$

$=(n-2)+(n-3)+...+2+1=\dfrac{(n-2)(n-1)}{2}$

65. $\lim\limits_{x\to 1}\dfrac{x^{100}-2x+1}{x^{50}-2x+1}.=\lim\limits_{x\to 1}\dfrac{(x-1)(x^{99}+x^{98}+...+x+1)-(x-1)}{(x-1)(x^{49}+x^{48}+...+x+1)-(x-1)}=\lim\limits_{x\to 1}\dfrac{x^{99}+x^{98}+...+x}{x^{49}+x^{48}+...+x}=\dfrac{49}{24}$

66. $\lim\limits_{x\to 1}\dfrac{x+x^2+...+x^n-n}{x-1}=\lim\limits_{x\to 1}\dfrac{(x-1)+(x^2-1)+...+(x^n-1)}{x-1}$

$=\lim\limits_{x\to 1}\dfrac{(x-1)+(x-1)(x+1)+...+(x-1)(x^{n-1}+x^{n-2}+...+x+1)}{x-1}$

$=\lim\limits_{x\to 1}(1+(x+1)+...+(x^{n-1}+x^{n-2}+...+x+1))\ =1+2+3+...+\mathrm{n}=\dfrac{n(n+1)}{2}$

Bài 2. **Tính các giới hạn sau:**

1. $\lim\limits_{x\to 1}\dfrac{\sqrt{x+3}-2}{x-1}$ ĐS: $\dfrac{1}{4}$.

2. $\lim\limits_{x\to -2}\dfrac{x+2}{\sqrt{x+3}-1}$ ĐS: 2

3. $\lim\limits_{x\to 6}\dfrac{3-\sqrt{x+3}}{x-6}$ ĐS: $-\dfrac{1}{6}$.

4. $\lim\limits_{x\to 8}\dfrac{x-8}{3-\sqrt{x+1}}$ ĐS: -6.

5. $\lim\limits_{x\to -1}\dfrac{\sqrt{4+x+x^2}-2}{x+1}$ ĐS: $-\dfrac{1}{4}$.

6. $\lim\limits_{x\to 3}\dfrac{\sqrt{x^2-3x}-x}{2x-6}$ ĐS: $\dfrac{1}{4}$.

7. $\lim\limits_{x\to 2}\dfrac{\sqrt{x+2}-2}{x^2-4}$ ĐS: $\dfrac{1}{16}$.

8. $\lim\limits_{x\to 2}\dfrac{2-\sqrt{3x-2}}{x^2-4}$ ĐS: $-\dfrac{3}{16}$.

9. $\lim\limits_{x\to 3}\dfrac{x^2-9}{\sqrt{x+1}-2}$ ĐS: 24.

10. $\lim\limits_{x\to 9}\dfrac{\sqrt{x}-3}{9x-x^2}$ ĐS: $-\dfrac{1}{54}$.

11. $\lim\limits_{x\to 7}\dfrac{x^2-49}{2-\sqrt{x-3}}$ ĐS: -56.

12. $\lim\limits_{x\to 1}\dfrac{2x-\sqrt{x+3}}{x^2-1}$ ĐS: $\dfrac{7}{8}$.

13. $\lim\limits_{x\to 3}\dfrac{x-\sqrt{3+2x}}{x^2-3x}$ ĐS: $\dfrac{2}{9}$.

14. $\lim\limits_{x\to 1}\dfrac{x^2-x}{\sqrt{2x-x^2}-1}$ ĐS: ∞.

15. $\lim\limits_{x\to 2}\dfrac{\sqrt{4x+1}-3}{x^2-2x}$ ĐS: $\dfrac{1}{3}$.

16. $\lim\limits_{x\to 4}\dfrac{\sqrt{3x-3}-3}{x^2-4x}$ ĐS: $\dfrac{1}{8}$.

17. $\lim\limits_{x\to 2}\dfrac{\sqrt{x+2}-2}{2x^2+x-10}$ ĐS: $\dfrac{1}{4}$.

18. $\lim\limits_{x\to 2}\dfrac{x^2-3x+2}{\sqrt{x-1}-1}$ ĐS: 2.

19. $\lim\limits_{x\to 4}\dfrac{x^2-3x-4}{\sqrt{x+5}-3}$ ĐS: 30.

20. $\lim\limits_{x\to 1}\dfrac{\sqrt{3x+1}-2}{x^2+x-2}$ ĐS: $\dfrac{1}{4}$.

21. $\lim\limits_{x\to 1}\dfrac{\sqrt{x}-1}{x^2-1}$ ĐS: $\dfrac{1}{4}$.

22. $\lim\limits_{x\to 2}\dfrac{3x^2-3(x+1)}{3-\sqrt{4x+1}}$ ĐS: -12.

23. $\lim\limits_{x\to 0}\dfrac{\sqrt{x^3+1}-1}{x^2+x}$ ĐS: 0.

24. $\lim\limits_{x\to -2}\dfrac{x+2}{\sqrt{1-x^3}-3}$ ĐS: $-\dfrac{1}{2}$.

25. $\lim\limits_{x\to 1}\dfrac{\sqrt{2x-x^2}-1}{x^2-x}$ ĐS: 0.

26. $\lim\limits_{x\to 2}\dfrac{\sqrt{2x+5}+x-5}{x^2-2x}$ ĐS: $\dfrac{2}{3}$.

27. $\lim\limits_{x\to 1}\dfrac{x^2-x}{\sqrt{2x+7}+x-4}$ ĐS: $\dfrac{3}{4}$.

28. $\lim\limits_{x\to -1}\dfrac{x-2+\sqrt{7-2x}}{x^2-1}$ ĐS: $\dfrac{1}{6}$.

29. $\lim\limits_{x\to -1}\dfrac{2x+5-\sqrt{2x^2+x+8}}{x^2+3x+2}$ ĐS: $\dfrac{5}{2}$.

30. $\lim\limits_{x\to 2}\dfrac{\sqrt{5x-6}-\sqrt{x+2}}{x-2}$ ĐS: 1.

31. $\lim\limits_{x\to -1}\dfrac{3x+3}{\sqrt{3+2x}-\sqrt{x+2}}$ ĐS: 6.

32. $\lim\limits_{x\to 3}\dfrac{\sqrt{x^2-2x+6}-\sqrt{x^2+2x-6}}{x^2-4x+3}$ ĐS: $-\dfrac{1}{3}$.

33. $\lim\limits_{x\to -1}\dfrac{\sqrt{x^2+x+2}-\sqrt{1-x}}{x^4+4}$ ĐS: 0.

34. $\lim\limits_{x\to 2}\dfrac{2-\sqrt{x+2}}{\sqrt{x+7}-3}$ ĐS: $-\dfrac{3}{2}$.

35. $\lim\limits_{x\to 9}\dfrac{3-\sqrt{x}}{\sqrt{x-5}-2}$ ĐS: $-\dfrac{2}{3}$.

36. $\lim\limits_{x\to 1}\dfrac{\sqrt{3x+1}-\sqrt{x+3}}{\sqrt{x+8}-3}$ ĐS: 3.

37. $\lim\limits_{x\to 2}\dfrac{\sqrt{x+2}-\sqrt{2x}}{\sqrt{x-1}-\sqrt{3-x}}$ ĐS: $-\dfrac{1}{4}$.

38. $\lim\limits_{x\to 1}\dfrac{\sqrt{x+3}-2}{\sqrt{4x+5}-\sqrt{3x+6}}$ ĐS: $\dfrac{3}{2}$.

39. $\lim\limits_{x\to 3}\dfrac{\sqrt{x+1}-\sqrt{3x-5}}{\sqrt{2x+3}-\sqrt{x+6}}$ ĐS: -3.

40. $\lim\limits_{x\to 2}\dfrac{\sqrt{2x^2+1}-\sqrt{2x+5}}{\sqrt{x^2+1}-\sqrt{x+3}}$ ĐS: $\dfrac{2\sqrt{5}}{3}$.

41. $\lim\limits_{x\to 1}\dfrac{x-1}{\sqrt{x+3}+x^2-3x}$ ĐS: $-\dfrac{4}{3}$.

42. $\lim\limits_{x\to 1}\dfrac{\sqrt[4]{4x+3}-1}{x-1}$ ĐS: 1.

43. $\lim\limits_{x\to 2}\dfrac{\sqrt{x-1}+x^4-3x^3+x^2+3}{\sqrt{2x}-2}$ ĐS: 1.

Lời giải

1. Ta có $\lim\limits_{x\to 1}\dfrac{\sqrt{x+3}-2}{x-1}=\lim\limits_{x\to 1}\dfrac{(\sqrt{x+3}-2)(\sqrt{x+3}+2)}{(x-1)(\sqrt{x+3}+2)}=\lim\limits_{x\to 1}\dfrac{1}{\sqrt{x+3}+2}=\dfrac{1}{4}$.

2. Ta có $\lim\limits_{x\to -2}\dfrac{x+2}{\sqrt{x+3}-1}=\lim\limits_{x\to -2}\dfrac{(x+2)(\sqrt{x+3}+1)}{(\sqrt{x+3}-1)(\sqrt{x+3}+1)}=\lim\limits_{x\to -2}(\sqrt{x+3}+1)=2$.

3. Ta có $\lim\limits_{x\to 6}\dfrac{3-\sqrt{x+3}}{x-6}=\lim\limits_{x\to 6}\dfrac{(3-\sqrt{x+3})(3+\sqrt{x+3})}{(x-6)(3+\sqrt{x+3})}=\lim\limits_{x\to 6}\dfrac{-1}{3+\sqrt{x+3}}=-\dfrac{1}{6}$

4. Ta có $\lim\limits_{x\to 8}\dfrac{x-8}{3-\sqrt{x+1}}=\lim\limits_{x\to 8}\dfrac{(x-8)(3+\sqrt{x+1})}{(3-\sqrt{x+1})(3+\sqrt{x+1})}=\lim\limits_{x\to 8}\dfrac{3+\sqrt{x+1}}{-1}=-6$.

5. Ta có $\lim\limits_{x\to -1}\dfrac{\sqrt{4+x+x^2}-2}{x+1}=\lim\limits_{x\to -1}\dfrac{x(x+1)}{(x+1)(\sqrt{4+x+x^2}-2)}=\lim\limits_{x\to -1}\dfrac{x}{\sqrt{4+x+x^2}-2}=-\dfrac{1}{4}$.

6. Ta có $\lim\limits_{x\to 3}\dfrac{\sqrt{2x^2-3x}-x}{2x-6}=\lim\limits_{x\to 3}\dfrac{(\sqrt{2x^2-3x}-x)(\sqrt{2x^2-3x}+x)}{(2x-6)(\sqrt{2x^2-3x}+x)}$

$=\lim\limits_{x\to 3}\dfrac{x(x-3)}{2(x-3)(\sqrt{2x^2-3x}+x)}=\lim\limits_{x\to 3}\dfrac{x}{2(\sqrt{2x^2-3x}+x)}=\dfrac{1}{4}$.

7. Ta có $\lim\limits_{x\to 2}\dfrac{\sqrt{2+x}-2}{x^2-4}=\lim\limits_{x\to 2}\dfrac{x-2}{(x-2)(x+2)(\sqrt{2+x}-2)}=\lim\limits_{x\to 2}\dfrac{1}{(x+2)(\sqrt{2+x}-2)}=\dfrac{1}{16}$.

8. Ta có $\lim\limits_{x\to 2}\dfrac{2-\sqrt{3x-2}}{x^2-4}=\lim\limits_{x\to 2}\dfrac{3(2-x)}{(x-2)(x+2)(2+\sqrt{3x-2})}=\lim\limits_{x\to 2}\dfrac{3}{(x+2)(2+\sqrt{3x-2})}=-\dfrac{3}{16}$.

9. Ta có $\lim\limits_{x\to 3}\dfrac{x^2-9}{\sqrt{x+1}-2}=\lim\limits_{x\to 3}\dfrac{(x+3)(x-3)(\sqrt{x+1}+2)}{(\sqrt{x+1}-2)(\sqrt{x+1}+2)}=\lim\limits_{x\to 3}\left[(x+3)(\sqrt{x+1}+2)\right]=24$.

10. Ta có $\lim\limits_{x\to 9}\dfrac{\sqrt{x}-3}{9x-x^2}=\lim\limits_{x\to 9}\dfrac{x-9}{x(9-x)(\sqrt{x}+3)}=\lim\limits_{x\to 9}\dfrac{-1}{x(\sqrt{x}+3)}=-\dfrac{1}{54}$.

11. Ta có $\lim\limits_{x\to 7}\dfrac{x^2-49}{2-\sqrt{x-3}}=\lim\limits_{x\to 7}\dfrac{(x-7)(x+7)(2+\sqrt{x-3})}{(2-\sqrt{x-3})(2+\sqrt{x-3})}$

$=\lim\limits_{x\to 7}\dfrac{(x-7)(x+7)(2+\sqrt{x-3})}{7-x}=-\lim\limits_{x\to 7}(x+7)(2+\sqrt{x-3})=-56$

12. Ta có $\lim\limits_{x\to 1}\dfrac{2x-\sqrt{x+3}}{x^2-1}=\lim\limits_{x\to 1}\dfrac{4x^2-x-3}{(x-1)(x+1)(2x+\sqrt{x+3})}=\lim\limits_{x\to 1}\dfrac{4x+3}{(x+1)(2x+\sqrt{x+3})}=\dfrac{7}{8}$.

13. Ta có $\lim\limits_{x\to 3}\dfrac{x-\sqrt{3+2x}}{x^2-3x}=\lim\limits_{x\to 3}\dfrac{x^2-2x-3}{x(x-3)(x+\sqrt{3+2x})}=\lim\limits_{x\to 3}\dfrac{x+1}{x(x+\sqrt{3+2x})}=\dfrac{2}{9}$.

14. Ta có $\lim\limits_{x\to 1}\dfrac{x^2-x}{\sqrt{2x-x^2}-1}=\lim\limits_{x\to 1}\dfrac{x(x-1)(\sqrt{2x-x^2}+1)}{-x^2+2x-1}=\lim\limits_{x\to 1}\dfrac{x(\sqrt{2x-x^2}+1)}{-(x-1)}=\infty$.

15. Ta có $\lim\limits_{x\to 2}\dfrac{\sqrt{4x+1}-3}{x^2-2x}=\lim\limits_{x\to 2}\dfrac{4(x-2)}{x(x-2)(\sqrt{4x+1}+3)}=\lim\limits_{x\to 2}\dfrac{4}{x(\sqrt{4x+1}+3)}=\dfrac{1}{3}$.

16. Ta có $\lim\limits_{x\to 4}\dfrac{\sqrt{3x-3}-3}{x^2-4x}=\lim\limits_{x\to 4}\dfrac{3(x-4)}{x(x-4)(\sqrt{3x-3}+3)}=\lim\limits_{x\to 4}\dfrac{3}{x(\sqrt{3x-3}+3)}=\dfrac{1}{8}$.

17. Ta có $\lim\limits_{x\to 2}\dfrac{\sqrt{x+2}-2}{2x^2+x-10}=\lim\limits_{x\to 2}\dfrac{(\sqrt{x+2}-2)(\sqrt{x+2}+2)}{(x-2)(2x-5)(\sqrt{x+2}+2)}=\lim\limits_{x\to 2}\dfrac{x-2}{(2x-5)(x-2)(\sqrt{x+2}+2)}$

$=\lim\limits_{x\to 2}\dfrac{1}{(2x-5)(\sqrt{x+2}+2)}=-\dfrac{1}{4}$

18. Ta có $\lim\limits_{x\to 2}\dfrac{x^2-3x+2}{\sqrt{x-1}-1}=\lim\limits_{x\to 2}\dfrac{(x-1)(x-2)(\sqrt{x-1}+1)}{(\sqrt{x-1}-1)(\sqrt{x-1}+1)}=\lim\limits_{x\to 2}(x-1)(\sqrt{x-1}+1)=2$.

19. Ta có $\lim\limits_{x\to 4}\dfrac{x^2-3x-4}{\sqrt{x+5}-3}=\lim\limits_{x\to 4}\dfrac{(x+1)(x-4)(\sqrt{x+5}+3)}{x-4}=\lim\limits_{x\to 4}(x+1)(\sqrt{x+5}+3)=30$.

20. Ta có $\lim\limits_{x\to 1}\dfrac{\sqrt{3x+1}-2}{x^2+x-2}=\lim\limits_{x\to 1}\dfrac{3(x-1)}{(x-1)(x+2)(\sqrt{3x+1}+2)}=\lim\limits_{x\to 1}\dfrac{3}{(x+2)(\sqrt{3x+1}+2)}=\dfrac{1}{4}$.

21. Ta có $\lim\limits_{x\to 1}\dfrac{\sqrt{x}-1}{x^2-1}=\lim\limits_{x\to 1}\dfrac{x-1}{(x+1)(x-1)(\sqrt{x}+1)}=\lim\limits_{x\to 1}\dfrac{1}{(x+1)(\sqrt{x}+1)}=\dfrac{1}{4}$.

22. Ta có $\lim\limits_{x\to 2}\dfrac{3x^2-4(x+1)}{3-\sqrt{4x+1}}=\lim\limits_{x\to 2}\dfrac{(x-2)(3x+2)(3+\sqrt{4x+1})}{(3-\sqrt{4x+1})(3+\sqrt{4x+1})}=\lim\limits_{x\to 2}\dfrac{(x-2)(3x+2)(3+\sqrt{4x+1})}{4(2-x)}$

$=\lim\limits_{x\to 2}\dfrac{(3x+2)(3+\sqrt{4x+1})}{4}=-12$.

23. Ta có $\lim\limits_{x\to 0}\dfrac{\sqrt{x^3+1}-1}{x^2+x}=\lim\limits_{x\to 0}\dfrac{x^3}{x(x+1)(\sqrt{x^3+1}+1)}=\lim\limits_{x\to 0}\dfrac{x^2}{(x+1)(\sqrt{x^3+1}+1)}=0$.

24. Ta có $\lim\limits_{x\to -2}\dfrac{x+2}{\sqrt{1-x^3}-3}=\lim\limits_{x\to -2}\dfrac{(x+2)(\sqrt{1-x^3}+3)}{-(x+2)(x^2-2x+4)}=\lim\limits_{x\to -2}\dfrac{\sqrt{1-x^3}+3}{-(x^2-2x+4)}=\dfrac{1}{2}$.

25. Ta có $\lim\limits_{x\to 1}\dfrac{\sqrt{2x-x^2}-1}{x^2-x}=\lim\limits_{x\to 1}\dfrac{-(x-1)^2}{x(x-1)(\sqrt{2x-x^2}+1)}=\lim\limits_{x\to 1}\dfrac{-(x-1)}{x(\sqrt{2x-x^2}+1)}=0$

26. Ta có $\lim\limits_{x\to 2}\dfrac{\sqrt{2x+5}+x-5}{x^2-2x}=\lim\limits_{x\to 2}\dfrac{-x^2+12x-20}{x(x-2)(\sqrt{2x+5}-(x+5))}$

$=\lim\limits_{x\to 2}\dfrac{-(x-2)(x-10)}{x(x-2)(\sqrt{2x+5}-(x+5))}=\lim\limits_{x\to 2}\dfrac{-(x-10)}{x(\sqrt{2x+5}-(x+5))}=\dfrac{2}{3}$

27. Ta có $\lim\limits_{x\to 1}\dfrac{x^2-x}{\sqrt{2x+7}+x-4}=\lim\limits_{x\to 1}\dfrac{x(x-1)(\sqrt{2x+7}-(x-4)}{-x^2+10x-9}=\lim\limits_{x\to 1}\dfrac{x(\sqrt{2x+7}-(x-4)}{-(x-9)}=\dfrac{3}{4}$

28. Ta có $\lim\limits_{x\to -1}\dfrac{x-2+\sqrt{7-2x}}{x^2-1}=\lim\limits_{x\to -1}\dfrac{x^2-2x-3}{(x-1)(x+1)((x-2)-\sqrt{7-2x})}$

$=\lim\limits_{x\to -1}\dfrac{x+3}{(x-1)((x-2)-\sqrt{7-2x})}=\dfrac{1}{6}$

29. Ta có $\lim\limits_{x\to -1}\dfrac{2x+5-\sqrt{2x^2+x+8}}{x^2+3x+2}=\lim\limits_{x\to -1}\dfrac{2x+17}{(x+2)((2x+5)+\sqrt{2x^2+x+8})}=\dfrac{5}{2}$

30. Ta có $\lim\limits_{x\to 2}\dfrac{\sqrt{5x-6}-\sqrt{x+2}}{x-2}=\lim\limits_{x\to 2}\dfrac{4(x-2)}{(x-2)(\sqrt{5x-6}+\sqrt{x+2})}=\lim\limits_{x\to 2}\dfrac{4}{(\sqrt{5x-6}+\sqrt{x+2})}=1$.

31. Ta có $\lim\limits_{x\to -1}\dfrac{3x+3}{\sqrt{3+2x}-\sqrt{x+2}}=\lim\limits_{x\to -1}\dfrac{3(x+1)(\sqrt{3+2x}+\sqrt{x+2})}{x+1}=\lim\limits_{x\to -1}3(\sqrt{3+2x}+\sqrt{x+2})=6$.

32. Ta có $\lim\limits_{x\to 3}\dfrac{\sqrt{x^2-2x+6}-\sqrt{x^2+2x-6}}{x^2-4x+3}=\lim\limits_{x\to 3}\dfrac{-4}{(x-1)(\sqrt{x^2-2x+6}+\sqrt{x^2+2x-6})}=\dfrac{1}{3}$.

33. Ta có $\lim\limits_{x\to -1}\dfrac{\sqrt{x^2+x+2}-\sqrt{1-x}}{x^4+x}=\lim\limits_{x\to -1}\dfrac{x+1}{x(x^2-x+1)(\sqrt{x^2+x+2}+\sqrt{1-x})}=0$.

34. Ta có $\lim\limits_{x\to 2}\dfrac{2-\sqrt{x+2}}{\sqrt{x+7}-3}=-\lim\limits_{x\to 2}\dfrac{\sqrt{x+7}+3}{2+\sqrt{x+2}}=-\dfrac{3}{2}$.

35. Ta có $\lim\limits_{x\to 9}\dfrac{3-\sqrt{x}}{\sqrt{x-5}-2}=-\lim\limits_{x\to 9}\dfrac{\sqrt{x-5}+2}{3+\sqrt{x}}=-\dfrac{2}{3}$.

36. Ta có $\lim\limits_{x\to 1}\dfrac{\sqrt{3x+1}-\sqrt{x+3}}{\sqrt{x+8}-3}=\lim\limits_{x\to 1}\dfrac{2(\sqrt{x+8}+3)}{\sqrt{3x+1}+\sqrt{x+3}}=3$.

37. Ta có $\lim\limits_{x\to 2}\dfrac{\sqrt{x+2}-\sqrt{2x}}{\sqrt{x-1}-\sqrt{3-x}}=\lim\limits_{x\to 2}\dfrac{\sqrt{x-1}+\sqrt{3-x}}{\sqrt{x+2}+\sqrt{2x}}=-\dfrac{1}{4}$.

38. Ta có $\lim\limits_{x\to 1}\dfrac{\sqrt{x+3}-2}{\sqrt{4x+5}-\sqrt{3x+6}}=\lim\limits_{x\to 1}\dfrac{\sqrt{4x+5}+\sqrt{3x+6}}{\sqrt{x+3}+2}=\dfrac{3}{2}$.

39. Ta có $\lim\limits_{x\to 3}\dfrac{\sqrt{x+1}-\sqrt{3x+5}}{\sqrt{2x+3}-\sqrt{x+6}}=-2\lim\limits_{x\to 3}\dfrac{\sqrt{2x+3}+\sqrt{x+6}}{\sqrt{x+1}+\sqrt{3x+5}}=-3$.

40. Ta có $\lim\limits_{x\to 2}\dfrac{\sqrt{2x^2+1}-\sqrt{2x+5}}{\sqrt{x^2+1}-\sqrt{x+3}}=\lim\limits_{x\to 2}2\dfrac{\sqrt{x^2+1}+\sqrt{x+3}}{\sqrt{2x^2+1}+\sqrt{2x+5}}=\dfrac{2\sqrt{5}}{3}$.

41. Ta có $\lim\limits_{x\to 1}\dfrac{x-1}{\sqrt{x+3}+x^2-3x}=\lim\limits_{x\to 1}\dfrac{\sqrt{x+3}-(x^2-3x)}{-x^3+5x^2-4x-3}=-\dfrac{4}{3}$.

42. Ta có $\lim\limits_{x\to 1}\dfrac{\sqrt[4]{4x-3}-1}{x-1}=\lim\limits_{x\to 1}\dfrac{4}{\sqrt[4]{(4x-3)^3}+\sqrt[4]{(4x-3)^2}+\sqrt[4]{4x-3}+1}=1$.

43. Ta có $\lim\limits_{x\to 2}\dfrac{\sqrt{x-1}+x^4-3x^3+x^2+3}{\sqrt{2x}-2}=1$.

Bài 3. Tính các giới hạn sau:

1. $\lim\limits_{x\to 2}\dfrac{\sqrt[3]{4x}-2}{x-2}$. ĐS: $\dfrac{1}{3}$.

2. $\lim\limits_{x\to -1}\dfrac{\sqrt[3]{5x-3}+2}{x+1}$ ĐS: $\dfrac{5}{12}$.

3. $\lim\limits_{x\to 0}\dfrac{1-\sqrt[3]{1-x}}{x^2+x}$ ĐS: $\dfrac{1}{3}$.

4. $\lim\limits_{x\to 1}\dfrac{2-\sqrt[3]{5x+3}}{x-1}$ ĐS: $-\dfrac{5}{12}$.

5. $\lim\limits_{x\to 3}\dfrac{x-3}{\sqrt[3]{x^2-1}-2}$ ĐS: 2.

6. $\lim\limits_{x\to 1}\dfrac{x-1}{1+\sqrt[3]{x-2}}$ ĐS: 3.

7. $\lim\limits_{x\to 1}\dfrac{\sqrt[3]{5x-4}-x}{2x^2-x-1}$ ĐS: $\dfrac{2}{9}$.

8. $\lim\limits_{x\to 1}\dfrac{x-1}{\sqrt[3]{x}-1}$ ĐS: 3.

9. $\lim\limits_{x\to 3}\dfrac{x^3-27}{x+1-\sqrt[3]{4x^2+28}}$ ĐS: 54.

10. $\lim\limits_{x\to 3}\dfrac{\sqrt[3]{x+5}-2}{x^3+x-30}$ ĐS: $\dfrac{1}{336}$.

11. $\lim\limits_{x\to -1}\dfrac{\sqrt[3]{10+2x^3}+x-1}{x^2+3x+2}$. ĐS: $\dfrac{3}{2}$.

12. $\lim\limits_{x\to 1}\dfrac{\sqrt[3]{x}-1}{\sqrt{x^2++3}-2}$ ĐS: $\dfrac{2}{3}$.

13. $\lim\limits_{x\to 1}\dfrac{\sqrt{x}-1}{\sqrt[3]{x+7}-2}$. ĐS: 6.

14. $\lim\limits_{x\to -1}\dfrac{\sqrt{x^2+3}-2}{\sqrt[3]{x}+1}$ ĐS: $-\dfrac{3}{2}$.

15. $\lim\limits_{x\to 1}\dfrac{\sqrt[3]{2x-1}-\sqrt[3]{x}}{\sqrt{x}-1}$. ĐS: $\dfrac{2}{3}$.

16. $\lim\limits_{x\to 1}\dfrac{\sqrt[3]{x}-1}{\sqrt{x^2-2}+1}$ ĐS: 1.

17. $\lim\limits_{x\to 1}\dfrac{\sqrt[3]{x}-1}{\sqrt[3]{4x+4}-2}$.	ĐS: 1.	18. $\lim\limits_{x\to -1}\dfrac{\sqrt[3]{x+2}+\sqrt[3]{x}}{x^2-1}$	ĐS: $-\dfrac{1}{3}$.
19. $\lim\limits_{x\to 1}\dfrac{\sqrt[3]{x+9}+\sqrt[3]{2x-6}}{x^3+1}$.	ĐS: $\dfrac{1}{12}$.	20. $\lim\limits_{x\to 3}\dfrac{\sqrt[3]{19-x^3}+2}{\sqrt{4x-3}-3}$	ĐS: $-\dfrac{27}{8}$.
21. $\lim\limits_{x\to 0}\dfrac{\sqrt[3]{1+x}-\sqrt[3]{1-x}}{x^2-4x}$.	ĐS: $-\dfrac{1}{6}$.	22. $\lim\limits_{x\to 1}\dfrac{\sqrt[3]{2x-1}-1}{x^3-1}$	ĐS: $\dfrac{2}{9}$.
23. $\lim\limits_{x\to 2}\dfrac{\sqrt[3]{3x+2}-x}{\sqrt{3x-2}-2}$.	ĐS: -1.	24. $\lim\limits_{x\to 0}\dfrac{\sqrt[3]{x+1}-1}{\sqrt[4]{2x+1}-1}$	ĐS: $\dfrac{2}{3}$.

Lời giải

1) Ta có $\lim\limits_{x\to 2}\dfrac{\sqrt[3]{4x}-2}{x-2}=\lim\limits_{x\to 2}\dfrac{4}{\sqrt[3]{16x^2}+2\sqrt[3]{4x}+4}=\dfrac{1}{3}$.

2) Ta có $\lim\limits_{x\to -1}\dfrac{\sqrt[3]{5x-3}+2}{x+1}=\lim\limits_{x\to -1}\dfrac{5}{\sqrt[3]{(5x-3)^2}-2\sqrt[3]{5x-3}+4}=\dfrac{5}{12}$.

3) Ta có $\lim\limits_{x\to 0}\dfrac{1-\sqrt[3]{1-x}}{x^2+x}=\lim\limits_{x\to 0}\dfrac{1}{(x+1)\left(1+\sqrt[3]{1-x}+\sqrt[3]{(1-x)^2}\right)}=\dfrac{1}{3}$.

4) Ta có $\lim\limits_{x\to 1}\dfrac{2-\sqrt[3]{5x+3}}{x-1}=\lim\limits_{x\to 1}\dfrac{-5}{4+2\sqrt[3]{5x+3}+\sqrt[3]{(5x+3)^2}}=-\dfrac{5}{12}$.

5) Ta có $\lim\limits_{x\to 3}\dfrac{x-3}{\sqrt[3]{x^2-1}-2}=\lim\limits_{x\to 3}\dfrac{\sqrt[3]{(x^2-1)^2}+2\sqrt[3]{x^2-1}+4}{x+3}=2$.

6) Ta có $\lim\limits_{x\to 1}\dfrac{x-1}{1+\sqrt[3]{x-2}}=\lim\limits_{x\to 1}\left(1-\sqrt[3]{x-2}+\sqrt[3]{(x-2)^2}\right)=3$.

7) Ta có $\lim\limits_{x\to 1}\dfrac{\sqrt[3]{5x-4}-x}{2x^2-x-1}=\lim\limits_{x\to 1}\dfrac{-x^2-x+4}{(2x+1)\left(\sqrt[3]{(5x-4)^2}+\sqrt[3]{5x-4}+4\right)}=\dfrac{2}{9}$.

8) Ta có $\lim\limits_{x\to 1}\dfrac{x-1}{\sqrt[3]{x}-1}=\lim\limits_{x\to 1}\left(\sqrt[3]{x^2}+\sqrt[3]{x}+1\right)=3$.

9) Ta có $\lim\limits_{x\to 3}\dfrac{x^3-27}{x+1-\sqrt[3]{4x^2+28}}$

$$=\lim\limits_{x\to 3}\frac{(x-3)(x^2+3x+9)\left[(x+1)^2+(x+1)\sqrt[3]{4x^2+28}+\sqrt[3]{(4x^2+28)^2}\right]}{(x-3)(x^2+2x+9)}$$

$$=\lim\limits_{x\to 3}\frac{(x^2+3x+9)\left[(x+1)^2+(x+1)\sqrt[3]{4x^2+28}+\sqrt[3]{(4x^2+28)^2}\right]}{x^2+2x+9}=72.$$

10) Ta có $\lim\limits_{x\to 3}\dfrac{\sqrt[3]{x+5}-2}{x^3+x-30}=\lim\limits_{x\to 3}\dfrac{1}{(x^2+3x+10)\left[\sqrt[3]{(x+5)^2}+\sqrt[3]{x+5}+4\right]}=\dfrac{1}{336}.$

11) Ta có $\lim\limits_{x\to -1}\dfrac{\sqrt[3]{10+2x^3}+x-1}{x^2+3x+2}=\lim\limits_{x\to -1}\dfrac{3x^3-3x^2+3x+9}{(x+1)(x+2)\left[\sqrt[3]{(10+2x^3)^2}+(x-1)\sqrt[3]{10+2x^3}+(x-1)^2\right]}$

$$=\lim\limits_{x\to -1}\frac{3x^2-6x+9}{(x+2)\left[\sqrt[3]{(10+2x^3)^2}+(x-1)\sqrt[3]{10+2x^3}+(x-1)^2\right]}=\frac{3}{2}.$$

12) Ta có $\lim\limits_{x\to 1}\dfrac{\sqrt[3]{x}-1}{\sqrt{x^2+3}-2}=\lim\limits_{x\to 1}\dfrac{\sqrt{x^2+3}+2}{(x+1)\left(\sqrt[3]{x^2}+\sqrt[3]{x}+1\right)}=\dfrac{2}{3}.$

13) Ta có $\lim\limits_{x\to 1}\dfrac{\sqrt{x}-1}{\sqrt[3]{x+7}-2}=\lim\limits_{x\to 1}\dfrac{\sqrt[3]{(x+7)^2}+2\sqrt[3]{x+7}+4}{\sqrt{x}+1}=6.$

14) Ta có $\lim\limits_{x\to -1}\dfrac{\sqrt{x^2+3}-2}{\sqrt[3]{x}+1}=\lim\limits_{x\to -1}\dfrac{(x-1)\left(\sqrt[3]{x^2}-\sqrt[3]{x}+1\right)}{\sqrt{x^2+3}+2}=-\dfrac{3}{2}.$

15) Ta có $\lim\limits_{x\to 1}\dfrac{\sqrt[3]{2x-1}-\sqrt[3]{x}}{\sqrt{x}-1}=\lim\limits_{x\to 1}\dfrac{\sqrt{x}+1}{\sqrt[3]{(2x-1)^2}+\sqrt[3]{x(2x-1)}+\sqrt[3]{x^2}}=\dfrac{2}{3}.$

16) Ta có $\lim\limits_{x\to 1}\dfrac{\sqrt[3]{x}-1}{\sqrt[3]{x-2}+1}=\lim\limits_{x\to 1}\dfrac{\sqrt[3]{(x-2)^2}-\sqrt[3]{x-2}+1}{\sqrt[3]{x^2}+\sqrt[3]{x}+1}=1.$

17) Ta có $\lim\limits_{x\to 1}\dfrac{\sqrt[3]{x}-1}{\sqrt[3]{4x+4}-2}=\lim\limits_{x\to 1}\dfrac{\sqrt[3]{(4x+4)^2}+2\sqrt[3]{4x+4}+4}{4\left(\sqrt[3]{x^2}+\sqrt[3]{x}+1\right)}=1.$

18) Ta có $\lim\limits_{x\to -1}\dfrac{\sqrt[3]{x+2}+\sqrt[3]{x}}{x^2-1}=\lim\limits_{x\to -1}\dfrac{2}{(x-1)\left[\sqrt[3]{(x+2)^2}-\sqrt[3]{x(x+2)}+\sqrt[3]{x^2}\right]}=-\dfrac{1}{3}.$

19) Ta có $\lim\limits_{x\to -1}\dfrac{\sqrt[3]{x+9}+\sqrt[3]{2x-6}}{x^3+1}$

$=\lim\limits_{x\to -1}\dfrac{3}{\left(x^2-x+1\right)\left[\sqrt[3]{\left(x+9\right)^2}-\sqrt[3]{\left(x+9\right)\left(2x-6\right)}+\sqrt[3]{\left(2x+6\right)^2}\right]}=\dfrac{1}{2}.$

20) Ta có $\lim\limits_{x\to 3}\dfrac{\sqrt[3]{19-x^3}+2}{\sqrt{4x-3}-3}=\lim\limits_{x\to 3}\dfrac{\left(9-3x+x^2\right)\left(\sqrt{4x-3}+3\right)}{-4\left[\sqrt[3]{\left(19-x^3\right)^2}-2\sqrt[3]{19-x^3}+4\right]}=-\dfrac{27}{8}.$

21) Ta có $\lim\limits_{x\to 0}\dfrac{\sqrt[3]{1+x}-\sqrt[3]{1-x}}{x^2-4x}=\lim\limits_{x\to 0}\dfrac{2}{\left(x-4\right)\left[\sqrt[3]{\left(1+x\right)^2}+\sqrt[3]{1-x^2}+\sqrt[3]{\left(1-x\right)^2}\right]}=-\dfrac{1}{6}.$

22) Ta có $\lim\limits_{x\to 1}\dfrac{\sqrt[3]{2x-1}-1}{x^3-1}=\lim\limits_{x\to 1}\dfrac{2}{\left(x^2+x+1\right)\left[\sqrt[3]{\left(2x-1\right)^2}+\sqrt[3]{2x-1}+1\right]}=\dfrac{2}{9}.$

23) Ta có $\lim\limits_{x\to 2}\dfrac{\sqrt[3]{3x+2}-x}{\sqrt{3x-2}-2}=\lim\limits_{x\to 2}\dfrac{-\left(x+1\right)^2\left(\sqrt{3x-2}+2\right)}{3\left[\sqrt[3]{\left(3x+2\right)^2}+x\sqrt[3]{3x+2}+x^2\right]}=-1.$

24) Ta có $\lim\limits_{x\to 0}\dfrac{\sqrt[3]{x+1}-1}{\sqrt[4]{2x+1}-1}=\lim\limits_{x\to 0}\dfrac{\left(\sqrt[4]{2x+1}+1\right)\left(\sqrt{2x+1}+1\right)}{2\left[\sqrt[3]{\left(x+1\right)^2}+\sqrt[3]{x+1}+1\right]}=\dfrac{2}{3}.$

Bài 4. Tính các giới hạn sau:

1) $\lim\limits_{x\to 0}\dfrac{\sqrt{x+9}+\sqrt{x+16}-7}{x}$. **ĐS:** $\dfrac{7}{24}$

2) $\lim\limits_{x\to 1}\dfrac{\sqrt{2x+2}+\sqrt{5x+4}-5}{x-1}$. **ĐS:** $\dfrac{4}{3}$

3) $\lim\limits_{x\to 3}\dfrac{2\sqrt{x+6}+\sqrt{2x-2}-8}{x-3}$. **ĐS:** $\dfrac{5}{6}$

4) $\lim\limits_{x\to 0}\dfrac{2\sqrt{x+1}+\sqrt{x+4}-4}{x}$. **ĐS:** $\dfrac{5}{4}$

5) $\lim\limits_{x\to 2}\dfrac{x\sqrt{x+2}+\sqrt{x+7}-7}{x-2}$. **ĐS:** $\dfrac{8}{3}$

6) $\lim\limits_{x\to 2}\dfrac{2x\sqrt{x-1}+x^2-8}{x-2}$. **ĐS:** 8

7) $\lim\limits_{x\to 6}\dfrac{\left(5x-4\right)\sqrt{2x-3}+x-84}{x-6}$. **ĐS:** $\dfrac{74}{3}$

8) $\lim\limits_{x\to 0}\dfrac{\sqrt{1+2x}-\sqrt[3]{1+3x}}{x}$. **ĐS:** 0

9) $\lim\limits_{x\to 1}\dfrac{\sqrt[3]{x^3+7}-\sqrt{x^2+3}}{x-1}$. **ĐS:** $-\dfrac{1}{4}$

10) $\lim\limits_{x\to 2}\dfrac{\sqrt[3]{8x+11}-\sqrt{x+7}}{x^2-3x+2}$. **ĐS:** $\dfrac{7}{54}$

11) $\lim\limits_{x\to 0}\dfrac{2\sqrt{1+x}-\sqrt[3]{8-x}}{x}$. **ĐS:** $\dfrac{13}{12}$

12) $\lim\limits_{x\to 1}\dfrac{\sqrt[3]{3x^2+5}-\sqrt{x+3}}{x-1}$. **ĐS:** $\dfrac{1}{4}$

13) $\lim\limits_{x\to 1}\dfrac{\sqrt[3]{x+7}-\sqrt{5-x^2}}{x-1}$. **ĐS:** $\dfrac{7}{12}$

14) $\lim\limits_{x\to 2}\dfrac{\sqrt[3]{3x+2}-\sqrt{3x-2}}{x-2}$. **ĐS:** $-\dfrac{1}{2}$

15) $\lim\limits_{x\to 2}\dfrac{\sqrt[3]{3x+2}-\sqrt{5x-6}}{x-2}$. **ĐS:** -1

16) $\lim\limits_{x\to 2}\dfrac{\sqrt[3]{2x^2+4x+11}-\sqrt{x+7}}{x^2-4}$. **ĐS:** $\dfrac{5}{72}$.

17) $\lim\limits_{x\to 1}\dfrac{\sqrt{5-x^3}-\sqrt[3]{x^2+7}}{x^2-1}$. **ĐS:** $-\dfrac{11}{24}$

18) $\lim\limits_{x\to 2}\dfrac{3\sqrt[3]{4x^3-24}+\sqrt{x+2}-8\sqrt{2x-3}}{4-x^2}$. **ĐS:** $-\dfrac{17}{16}$

19) $\lim\limits_{x\to 1}\dfrac{\sqrt{3x-2}-\sqrt[3]{4x^2-x-2}}{x^2-3x+2}$. **ĐS:** $\dfrac{5}{6}$

20) $\lim\limits_{x\to 1}\dfrac{x\sqrt{2x-1}+\sqrt[3]{3x-2}-2}{x^2-1}$. **ĐS:** $\dfrac{3}{2}$

21) $\lim\limits_{x\to 0}\dfrac{\sqrt[3]{1+x^2}-\sqrt[4]{1-2x}}{x^2+x}$. **ĐS:** $\dfrac{1}{2}$

22) $\lim\limits_{x\to 2}\dfrac{\sqrt[3]{x+6}-\sqrt[4]{7x+2}}{x-2}$. **ĐS:** $-\dfrac{13}{96}$

23) $\lim\limits_{x\to 0}\dfrac{\sqrt{1+4x}.\sqrt{1+6x}-1}{x}$. **ĐS:** 5

24) $\lim\limits_{x\to 0}\dfrac{\sqrt{1+2x}.\sqrt[3]{1+4x}-1}{x}$. **ĐS:** $\dfrac{7}{3}$

25) $\lim\limits_{x\to 1}\dfrac{\sqrt{3x+1}.\sqrt[3]{2-x}-2}{x-1}$. **ĐS:** $\dfrac{1}{12}$

26) $\lim\limits_{x\to 0}\dfrac{\sqrt{4+x}.\sqrt[3]{8+3x}-4}{x^2+x}$. **ĐS:** 1

27) $\lim\limits_{x\to 0}\dfrac{\sqrt{4x+4}+\sqrt{9-6x}-5}{x^2}$. **ĐS:** $-\dfrac{5}{12}$

28) $\lim\limits_{x\to 0}\dfrac{\sqrt{1+2x}-\sqrt[3]{1+3x}}{x^2}$. **ĐS:** $\dfrac{1}{2}$

29) $\lim\limits_{x\to 1}\dfrac{\sqrt{6x+3}+2x^2-5x}{(x-1)^2}$. **ĐS:** $\dfrac{11}{6}$

30) $\lim\limits_{x\to 1}\dfrac{\sqrt{4x-3}+\sqrt{2x-1}-3x+1}{x^2-2x+1}$. **ĐS:** $-\dfrac{5}{2}$.

31) $\lim\limits_{x\to 1}\dfrac{-3x-7+4\sqrt{x+3}+2\sqrt{2x-1}}{x^2-2x+1}$. **ĐS:** $-\dfrac{17}{16}$

32) $\lim\limits_{x\to 2}\dfrac{x^2-4x+4}{\sqrt{2x^2+8}-2\sqrt{2x-3}+x-4}$. **ĐS:** $\dfrac{8}{9}$

33) $\lim\limits_{x\to 1}\dfrac{\sqrt[3]{6x^2+2}-2\sqrt{x}}{x^3-x^2-x+1}$. **ĐS:** $\dfrac{1}{8}$

34) $\lim\limits_{x\to 2}\dfrac{\sqrt{2x^2-6x+5}-\sqrt[3]{3x^2-9x+7}}{(x-2)^2}$ **ĐS:** $\dfrac{1}{2}$

35) $\lim\limits_{x\to 0}\dfrac{\sqrt{1+2x}-\sqrt[3]{1+3x}}{x^2}$. **ĐS:** $\dfrac{1}{2}$

36) $\lim\limits_{x\to 0}\dfrac{\sqrt{1+4x}-\sqrt[3]{1+6x}}{x^2}$. **ĐS:** 2

Lời giải

1) $I=\lim\limits_{x\to 0}\dfrac{\sqrt{x+9}+\sqrt{x+16}-7}{x}$.

Ta có $I=\lim\limits_{x\to 0}\left(\dfrac{\sqrt{x+9}-3}{x}+\dfrac{\sqrt{x+16}-4}{x}\right)$

$$=\lim_{x\to 0}\left[\frac{\left(\sqrt{x+9}-3\right)\left(\sqrt{x+9}+3\right)}{x\left(\sqrt{x+9}+3\right)}+\frac{\left(\sqrt{x+16}-4\right)\left(\sqrt{x+16}+4\right)}{x\left(\sqrt{x+16}+4\right)}\right]$$

$$=\lim_{x\to 0}\left[\frac{x+9-9}{x\left(\sqrt{x+9}+3\right)}+\frac{x+16-16}{x\left(\sqrt{x+16}+4\right)}\right]=\lim_{x\to 0}\left[\frac{x}{x\left(\sqrt{x+9}+3\right)}+\frac{x}{x\left(\sqrt{x+16}+4\right)}\right]$$

$$=\lim_{x\to 0}\left(\frac{1}{\sqrt{x+9}+3}+\frac{1}{\sqrt{x+16}+4}\right)=\frac{1}{6}+\frac{1}{8}=\frac{7}{24}.$$

2) $I = \lim\limits_{x\to 1}\dfrac{\sqrt{2x+2}+\sqrt{5x+4}-5}{x-1}$.

Ta có $I = \lim\limits_{x\to 1}\left(\dfrac{\sqrt{2x+2}-2}{x-1}+\dfrac{\sqrt{5x+4}-3}{x-1}\right)$

$$= \lim_{x\to 1}\left[\frac{\left(\sqrt{2x+2}-2\right)\left(\sqrt{2x+2}+2\right)}{(x-1)\left(\sqrt{2x+2}+2\right)}+\frac{\left(\sqrt{5x+4}-3\right)\left(\sqrt{5x+4}+3\right)}{(x-1)\left(\sqrt{5x+4}+3\right)}\right]$$

$$= \lim_{x\to 1}\left[\frac{2x+2-4}{(x-1)\left(\sqrt{2x+2}+2\right)}+\frac{5x+4-9}{(x-1)\left(\sqrt{5x+4}+3\right)}\right]$$

$$= \lim_{x\to 1}\left[\frac{2(x-1)}{(x-1)\left(\sqrt{2x+2}+2\right)}+\frac{5(x-1)}{(x-1)\left(\sqrt{5x+4}+3\right)}\right]$$

$$= \lim_{x\to 1}\left(\frac{2}{\sqrt{2x+2}+2}+\frac{5}{\sqrt{5x+4}+3}\right) = \frac{2}{4}+\frac{5}{6}=\frac{4}{3}.$$

3) $I = \lim\limits_{x\to 3}\dfrac{2\sqrt{x+6}+\sqrt{2x-2}-8}{x-3}$.

Ta có $I = \lim\limits_{x\to 3}\left(\dfrac{2\sqrt{x+6}-6}{x-3}+\dfrac{\sqrt{2x-2}-2}{x-3}\right)$

$$= \lim_{x\to 3}\left[\frac{2\left(\sqrt{x+6}-3\right)\left(\sqrt{x+6}+3\right)}{(x-3)\left(\sqrt{x+6}+3\right)}+\frac{\left(\sqrt{2x-2}-2\right)\left(\sqrt{2x-2}+2\right)}{(x-3)\left(\sqrt{2x-2}+2\right)}\right]$$

$$= \lim_{x\to 3}\left[\frac{2(x+6-9)}{(x-3)\left(\sqrt{x+6}+3\right)}+\frac{2x-2-4}{(x-3)\left(\sqrt{2x-2}+2\right)}\right]$$

$$= \lim_{x\to 3}\left[\frac{2(x-3)}{(x-3)\left(\sqrt{x+6}+3\right)}+\frac{2(x-3)}{(x-3)\left(\sqrt{2x-2}+2\right)}\right]$$

$$= \lim_{x\to 3}\left(\frac{2}{\sqrt{x+6}+3}+\frac{2}{\sqrt{2x-2}+2}\right)=\frac{2}{6}+\frac{2}{4}=\frac{5}{6}.$$

4) $I = \lim\limits_{x\to 0}\dfrac{2\sqrt{x+1}+\sqrt{x+4}-4}{x}$.

Ta có $I = \lim\limits_{x\to 0}\left(\dfrac{2\sqrt{x+1}-2}{x}+\dfrac{\sqrt{x+4}-2}{x}\right)$

$$= \lim_{x\to 0}\left[\frac{2\left(\sqrt{x+1}-1\right)\left(\sqrt{x+1}+1\right)}{x\left(\sqrt{x+1}+1\right)}+\frac{\left(\sqrt{x+4}-2\right)\left(\sqrt{x+4}+2\right)}{x\left(\sqrt{x+4}+2\right)}\right]$$

$$= \lim_{x\to 0}\left[\frac{2(x+1-1)}{x\left(\sqrt{x+1}+1\right)}+\frac{x+4-4}{x\left(\sqrt{x+4}+2\right)}\right] = \lim_{x\to 0}\left(\frac{2}{\sqrt{x+1}+1}+\frac{1}{\sqrt{x+4}+2}\right) = \frac{2}{2}+\frac{1}{4}=\frac{5}{4}$$

5) $I = \lim\limits_{x\to 2}\dfrac{x\sqrt{x+2}+\sqrt{x+7}-7}{x-2}$.

Ta có $I=\lim\limits_{x\to2}\dfrac{(x-2)\sqrt{x+2}+2\sqrt{x+2}-4+\sqrt{x+7}-3}{x-2}=\lim\limits_{x\to2}\left(\sqrt{x+2}+\dfrac{2\sqrt{x+2}-4}{x-2}+\dfrac{\sqrt{x+7}-3}{x-2}\right)$

$=2+\lim\limits_{x\to2}\left[\dfrac{2\left(\sqrt{x+2}-2\right)\left(\sqrt{x+2}+2\right)}{(x-2)\left(\sqrt{x+2}+2\right)}+\dfrac{\left(\sqrt{x+7}-3\right)\left(\sqrt{x+7}+3\right)}{(x-2)\left(\sqrt{x+7}+3\right)}\right]$

$=2+\lim\limits_{x\to2}\left(\dfrac{2}{\sqrt{x+2}+2}+\dfrac{1}{\sqrt{x+7}+3}\right)=2+\dfrac{2}{4}+\dfrac{1}{6}=\dfrac{8}{3}.$

6) $I=\lim\limits_{x\to2}\dfrac{2x\sqrt{x-1}+x^2-8}{x-2}.$

Ta có $I=\lim\limits_{x\to2}\dfrac{2(x-2)\sqrt{x-1}+4\sqrt{x-1}-4+x^2-4}{x-2}=\lim\limits_{x\to2}\left(2\sqrt{x-1}+\dfrac{4\sqrt{x-1}-4}{x-2}+\dfrac{x^2-4}{x-2}\right)$

$=2+\lim\limits_{x\to2}\left[\dfrac{4\left(\sqrt{x-1}-1\right)\left(\sqrt{x-1}+1\right)}{(x-2)\left(\sqrt{x+1}+1\right)}+\dfrac{(x-2)(x+2)}{x-2}\right]=2+\lim\limits_{x\to2}\left[\dfrac{4(x-1-1)}{(x-2)\left(\sqrt{x+1}+1\right)}+x+2\right]$

$=2+\lim\limits_{x\to2}\left(\dfrac{4}{\sqrt{x-1}+1}+x+2\right)=2+\dfrac{4}{2}+4=8.$

7) $I=\lim\limits_{x\to6}\dfrac{(5x-4)\sqrt{2x-3}+x-84}{x-6}.$

Ta có $I=\lim\limits_{x\to6}\dfrac{(5x-30)\sqrt{2x-3}+26\sqrt{2x-3}-78+x-6}{x-6}$

$=\lim\limits_{x\to6}\left[\dfrac{5(x-6)\sqrt{2x-3}}{x-6}+\dfrac{26\left(\sqrt{2x-3}-3\right)}{x-6}+\dfrac{x-6}{x-6}\right]$

$=\lim\limits_{x\to6}\left[5\sqrt{2x-3}+\dfrac{26\left(\sqrt{2x-3}-3\right)\left(\sqrt{2x-3}+3\right)}{(x-6)\left(\sqrt{2x-3}+3\right)}+1\right]$

$=15+\lim\limits_{x\to6}\dfrac{26(2x-3-9)}{(x-6)\left(\sqrt{2x-3}+3\right)}+1=15+\lim\limits_{x\to6}\dfrac{26.2(x-6)}{(x-6)\left(\sqrt{2x-3}+3\right)}+1$

$=15+\lim\limits_{x\to6}\dfrac{52}{\sqrt{2x-3}+3}+1=15+\dfrac{52}{6}+1=\dfrac{74}{3}.$

8) $I=\lim\limits_{x\to0}\dfrac{\sqrt{1+2x}-\sqrt[3]{1+3x}}{x}.$

Ta có $I=\lim\limits_{x\to0}\dfrac{\sqrt{1+2x}-1+1-\sqrt[3]{1+3x}}{x}=\lim\limits_{x\to0}\left(\dfrac{\sqrt{1+2x}-1}{x}+\dfrac{1-\sqrt[3]{1+3x}}{x}\right)$

$=\lim\limits_{x\to0}\left[\dfrac{\left(\sqrt{1+2x}-1\right)\left(\sqrt{1+2x}+1\right)}{x\left(\sqrt{1+2x}+1\right)}+\dfrac{\left(1-\sqrt[3]{1+3x}\right)\left(1+\sqrt[3]{1+3x}+\sqrt[3]{(1+3x)^2}\right)}{x\left(1+\sqrt[3]{1+3x}+\sqrt[3]{(1+3x)^2}\right)}\right]$

$$=\lim_{x\to 0}\left[\frac{1+2x-1}{x\left(\sqrt{1+2x}+1\right)}+\frac{1-(1+3x)}{x\left(1+\sqrt[3]{1+3x}+\sqrt[3]{(1+3x)^2}\right)}\right]=\lim_{x\to 0}\left(\frac{2}{\sqrt{1+2x}+1}+\frac{-3}{1+\sqrt[3]{1+3x}+\sqrt[3]{(1+3x)^2}}\right)$$

$$=\frac{2}{2}+\frac{-3}{3}=0.$$

9) $I=\lim\limits_{x\to 1}\dfrac{\sqrt[3]{x^3+7}-\sqrt{x^2+3}}{x-1}$.

Ta có $I=\lim\limits_{x\to 1}\dfrac{\sqrt[3]{x^3+7}-2+2-\sqrt{x^2+3}}{x-1}=\lim\limits_{x\to 1}\left(\dfrac{\sqrt[3]{x^3+7}-2}{x-1}+\dfrac{2-\sqrt{x^2+3}}{x-1}\right)$

$$=\lim_{x\to 1}\left[\frac{\left(\sqrt[3]{x^3+7}-2\right)\left(\sqrt[3]{(x^3+7)^2}+2\sqrt[3]{x^3+7}+4\right)}{(x-1)\left(\sqrt[3]{(x^3+7)^2}+2\sqrt[3]{x^3+7}+4\right)}+\frac{\left(2-\sqrt{x^2+3}\right)\left(2+\sqrt{x^2+3}\right)}{(x-1)\left(2+\sqrt{x^2+3}\right)}\right]$$

$$=\lim_{x\to 1}\left[\frac{x^3+7-8}{(x-1)\left(\sqrt[3]{(x^3+7)^2}+2\sqrt[3]{x^3+7}+4\right)}+\frac{4-(x^2+3)}{(x-1)\left(2+\sqrt{x^2+3}\right)}\right]$$

$$=\lim_{x\to 1}\left[\frac{x^3-1}{(x-1)\left(\sqrt[3]{(x^3+7)^2}+2\sqrt[3]{x^3+7}+4\right)}+\frac{1-x^2}{(x-1)\left(2+\sqrt{x^2+3}\right)}\right]$$

$$=\lim_{x\to 1}\left[\frac{x^2+x+1}{\sqrt[3]{(x^3+7)^2}+2\sqrt[3]{x^3+7}+4}-\frac{x+1}{2+\sqrt{x^2+3}}\right]=\frac{3}{12}-\frac{2}{4}=-\frac{1}{4}.$$

10) $I=\lim\limits_{x\to 2}\dfrac{\sqrt[3]{8x+11}-\sqrt{x+7}}{x^2-3x+2}$.

Ta có $I=\lim\limits_{x\to 2}\dfrac{\sqrt[3]{8x+11}-3+3-\sqrt{x+7}}{x^2-3x+2}=\lim\limits_{x\to 2}\left(\dfrac{\sqrt[3]{8x+11}-3}{x^2-3x+2}+\dfrac{3-\sqrt{x+7}}{x^2-3x+2}\right)$

$$=\lim_{x\to 2}\left[\frac{\left(\sqrt[3]{8x+11}-3\right)\left(\sqrt[3]{(8x+11)^2}+3\sqrt[3]{8x+11}+9\right)}{(x^2-3x+2)\left(\sqrt[3]{(8x+11)^2}+3\sqrt[3]{8x+11}+9\right)}+\frac{\left(3-\sqrt{x+7}\right)\left(3+\sqrt{x+7}\right)}{(x^2-3x+2)\left(3+\sqrt{x+7}\right)}\right]$$

$$=\lim_{x\to 2}\left[\frac{8x+11-27}{(x^2-3x+2)\left(\sqrt[3]{(8x+11)^2}+3\sqrt[3]{8x+11}+9\right)}+\frac{9-(x+7)}{(x^2-3x+2)\left(3+\sqrt{x+7}\right)}\right]$$

$$=\lim_{x\to2}\left[\frac{8(x-2)}{(x-1)(x-2)\left(\sqrt[3]{(8x+11)^2}+3\sqrt[3]{8x+11}+9\right)}+\frac{2-x}{(x-1)(x-2)\left(3+\sqrt{x+7}\right)}\right]$$

$$=\lim_{x\to2}\left[\frac{8}{(x-1)\left(\sqrt[3]{(8x+11)^2}+3\sqrt[3]{8x+11}+9\right)}-\frac{1}{(x-1)\left(3+\sqrt{x+7}\right)}\right]=\frac{8}{27}-\frac{1}{6}=\frac{7}{54}.$$

11) $I=\lim\limits_{x\to0}\dfrac{2\sqrt{1+x}-\sqrt[3]{8-x}}{x}$.

Ta có $I=\lim\limits_{x\to0}\dfrac{2\sqrt{1+x}-2+2-\sqrt[3]{8-x}}{x}=\lim\limits_{x\to0}\left(\dfrac{2\sqrt{1+x}-2}{x}+\dfrac{2-\sqrt[3]{8-x}}{x}\right)$

$$=\lim_{x\to0}\left[\frac{2\left(\sqrt{1+x}-1\right)\left(\sqrt{1+x}+1\right)}{x\left(\sqrt{1+x}+1\right)}+\frac{\left(2-\sqrt[3]{8-x}\right)\left(4+2\sqrt[3]{8-x}+\sqrt[3]{(8-x)^2}\right)}{x\left(4+2\sqrt[3]{8-x}+\sqrt[3]{(8-x)^2}\right)}\right]$$

$$=\lim_{x\to0}\left[\frac{2(1+x-1)}{x\left(\sqrt{1+x}+1\right)}+\frac{8-(8-x)}{x\left(4+2\sqrt[3]{8-x}+\sqrt[3]{(8-x)^2}\right)}\right]$$

$$=\lim_{x\to0}\left[\frac{2}{\sqrt{1+x}+1}+\frac{1}{4+2\sqrt[3]{8-x}+\sqrt[3]{(8-x)^2}}\right]=\frac{2}{2}+\frac{1}{12}=\frac{13}{12}.$$

12) $I=\lim\limits_{x\to1}\dfrac{\sqrt[3]{3x^2+5}-\sqrt{x+3}}{x-1}$.

Ta có $I=\lim\limits_{x\to1}\dfrac{\sqrt[3]{3x^2+5}-2+2-\sqrt{x+3}}{x-1}=\lim\limits_{x\to1}\left(\dfrac{\sqrt[3]{3x^2+5}-2}{x-1}+\dfrac{2-\sqrt{x+3}}{x-1}\right)$

$$=\lim_{x\to1}\left[\frac{\left(\sqrt[3]{3x^2+5}-2\right)\left(\sqrt[3]{(3x^2+5)^2}+2\sqrt[3]{3x^2+5}+4\right)}{(x-1)\left(\sqrt[3]{(3x^2+5)^2}+2\sqrt[3]{3x^2+5}+4\right)}+\frac{\left(2-\sqrt{x+3}\right)\left(2+\sqrt{x+3}\right)}{(x-1)\left(2+\sqrt{x+3}\right)}\right]$$

$$=\lim_{x\to1}\left[\frac{3x^2+5-8}{(x-1)\left(\sqrt[3]{(3x^2+5)^2}+2\sqrt[3]{3x^2+5}+4\right)}+\frac{4-(x+3)}{(x-1)\left(2+\sqrt{x+3}\right)}\right]$$

$$=\lim_{x\to1}\left[\frac{3\left(x^2-1\right)}{(x-1)\left(\sqrt[3]{(3x^2+5)^2}+2\sqrt[3]{3x^2+5}+4\right)}+\frac{1-x}{(x-1)\left(2+\sqrt{x+3}\right)}\right]$$

$$=\lim_{x\to 1}\left[\frac{3(x+1)}{\sqrt[3]{(3x^2+5)^2}+2\sqrt[3]{3x^2+5}+4}-\frac{1}{2+\sqrt{x+3}}\right]=\frac{6}{12}-\frac{1}{4}=\frac{1}{4}.$$

13) $I=\lim_{x\to 1}\dfrac{\sqrt[3]{x+7}-\sqrt{5-x^2}}{x-1}$.

Ta có $I=\lim_{x\to 1}\dfrac{\sqrt[3]{x+7}-2+2-\sqrt{5-x^2}}{x-1}=\lim_{x\to 1}\left(\dfrac{\sqrt[3]{x+7}-2}{x-1}+\dfrac{2-\sqrt{5-x^2}}{x-1}\right)$

$$=\lim_{x\to 1}\left[\frac{\left(\sqrt[3]{x+7}-2\right)\left(\sqrt[3]{(x+7)^2}+2\sqrt[3]{x+7}+4\right)}{(x-1)\left(\sqrt[3]{(x+7)^2}+2\sqrt[3]{x+7}+4\right)}+\frac{\left(2-\sqrt{5-x^2}\right)\left(2+\sqrt{5-x^2}\right)}{(x-1)\left(2+\sqrt{5-x^2}\right)}\right]$$

$$=\lim_{x\to 1}\left[\frac{x+7-8}{(x-1)\left(\sqrt[3]{(x+7)^2}+2\sqrt[3]{x+7}+4\right)}+\frac{4-(5-x^2)}{(x-1)\left(2+\sqrt{5-x^2}\right)}\right]$$

$$=\lim_{x\to 1}\left[\frac{x-1}{(x-1)\left(\sqrt[3]{(x+7)^2}+2\sqrt[3]{x+7}+4\right)}+\frac{x^2-1}{(x-1)\left(2+\sqrt{5-x^2}\right)}\right]$$

$$=\lim_{x\to 1}\left[\frac{1}{\sqrt[3]{(x+7)^2}+2\sqrt[3]{x+7}+4}+\frac{x+1}{2+\sqrt{5-x^2}}\right]=\frac{1}{12}+\frac{2}{4}=\frac{7}{12}.$$

14) $I=\lim_{x\to 2}\dfrac{\sqrt[3]{3x+2}-\sqrt{3x-2}}{x-2}$.

Ta có $I=\lim_{x\to 2}\dfrac{\sqrt[3]{3x+2}-2+2-\sqrt{3x-2}}{x-2}=\lim_{x\to 2}\left(\dfrac{\sqrt[3]{3x+2}-2}{x-2}+\dfrac{2-\sqrt{3x-2}}{x-2}\right)$

$$=\lim_{x\to 2}\left[\frac{\left(\sqrt[3]{3x+2}-2\right)\left(\sqrt[3]{(3x+2)^2}+2\sqrt[3]{3x+2}+4\right)}{(x-2)\left(\sqrt[3]{(3x+2)^2}+2\sqrt[3]{3x+2}+4\right)}+\frac{\left(2-\sqrt{3x-2}\right)\left(2+\sqrt{3x-2}\right)}{(x-2)\left(2+\sqrt{3x-2}\right)}\right]$$

$$=\lim_{x\to 2}\left[\frac{3x+2-8}{(x-2)\left(\sqrt[3]{(3x+2)^2}+2\sqrt[3]{3x+2}+4\right)}+\frac{4-(3x-2)}{(x-2)\left(2+\sqrt{3x-2}\right)}\right]$$

$$=\lim_{x\to 2}\left[\frac{3(x-2)}{(x-2)\left(\sqrt[3]{(3x+2)^2}+2\sqrt[3]{3x+2}+4\right)}+\frac{6-3x}{(x-2)\left(2+\sqrt{3x-2}\right)}\right]$$

$$=\lim_{x\to 2}\left[\frac{3}{\sqrt[3]{(3x+2)^2}+2\sqrt[3]{3x+2}+4}-\frac{3}{2+\sqrt{3x-2}}\right]=\frac{3}{12}+\frac{-3}{4}=-\frac{1}{2}.$$

15) $I=\lim\limits_{x\to 2}\dfrac{\sqrt[3]{3x+2}-\sqrt{5x-6}}{x-2}$.

Ta có $I=\lim\limits_{x\to 2}\dfrac{\sqrt[3]{3x+2}-2+2-\sqrt{5x-6}}{x-2}=\lim\limits_{x\to 2}\left(\dfrac{\sqrt[3]{3x+2}-2}{x-2}+\dfrac{2-\sqrt{5x-6}}{x-2}\right)$

$$=\lim_{x\to 2}\left[\frac{\left(\sqrt[3]{3x+2}-2\right)\left(\sqrt[3]{(3x+2)^2}+2\sqrt[3]{3x+2}+4\right)}{(x-2)\left(\sqrt[3]{(3x+2)^2}+2\sqrt[3]{3x+2}+4\right)}+\frac{\left(2-\sqrt{5x-6}\right)\left(2+\sqrt{5x-6}\right)}{(x-2)\left(2+\sqrt{5x-6}\right)}\right]$$

$$=\lim_{x\to 2}\left[\frac{3x+2-8}{(x-2)\left(\sqrt[3]{(3x+2)^2}+2\sqrt[3]{3x+2}+4\right)}+\frac{4-(5x-6)}{(x-2)\left(2+\sqrt{5x-6}\right)}\right]$$

$$=\lim_{x\to 2}\left[\frac{3(x-2)}{(x-2)\left(\sqrt[3]{(3x+2)^2}+2\sqrt[3]{3x+2}+4\right)}+\frac{10-5x}{(x-2)\left(2+\sqrt{5x-6}\right)}\right]$$

$$=\lim_{x\to 2}\left[\frac{3}{\sqrt[3]{(3x+2)^2}+2\sqrt[3]{3x+2}+4}-\frac{5}{2+\sqrt{5x-6}}\right]=\frac{3}{12}+\frac{-5}{4}=-1.$$

16) $I=\lim\limits_{x\to 2}\dfrac{\sqrt[3]{2x^2+4x+11}-\sqrt{x+7}}{x^2-4}$.

Ta có $I=\lim\limits_{x\to 2}\dfrac{\sqrt[3]{2x^2+4x+11}-3+3-\sqrt{x+7}}{x^2-4}=\lim\limits_{x\to 2}\left(\dfrac{\sqrt[3]{2x^2+4x+11}-3}{x^2-4}+\dfrac{3-\sqrt{x+7}}{x^2-4}\right)$

$$=\lim_{x\to 2}\left[\frac{\left(\sqrt[3]{2x^2+4x+11}-3\right)\left(\sqrt[3]{(2x^2+4x+11)^2}+3\sqrt[3]{2x^2+4x+11}+9\right)}{(x^2-4)\left(\sqrt[3]{(2x^2+4x+11)^2}+3\sqrt[3]{2x^2+4x+11}+9\right)}+\frac{\left(3-\sqrt{x+7}\right)\left(3+\sqrt{x+7}\right)}{(x^2-4)\left(3+\sqrt{x+7}\right)}\right]$$

$$=\lim_{x\to 2}\left[\frac{2x^2+4x+11-27}{(x^2-4)\left(\sqrt[3]{(2x^2+4x+11)^2}+3\sqrt[3]{2x^2+4x+11}+9\right)}+\frac{9-(x+7)}{(x^2-4)\left(3+\sqrt{x+7}\right)}\right]$$

$$=\lim_{x\to 2}\left[\frac{2x^2+4x-16}{(x^2-4)\left(\sqrt[3]{(2x^2+4x+11)^2}+3\sqrt[3]{2x^2+4x+11}+9\right)}+\frac{2-x}{(x^2-4)\left(3+\sqrt{x+7}\right)}\right]$$

$$=\lim_{x\to 2}\left[\frac{2(x-2)(x+4)}{(x^2-4)\left(\sqrt[3]{(2x^2+4x+11)^2}+3\sqrt[3]{2x^2+4x+11}+9\right)}+\frac{2-x}{(x^2-4)\left(3+\sqrt{x+7}\right)}\right]$$

$$=\lim_{x\to 2}\left[\frac{2(x+4)}{(x+2)\left(\sqrt[3]{(2x^2+4x+11)^2}+3\sqrt[3]{2x^2+4x+11}+9\right)}-\frac{1}{(x+2)\left(3+\sqrt{x+7}\right)}\right]=\frac{12}{108}+\frac{-1}{24}=\frac{5}{72}.$$

17) $I=\lim\limits_{x\to 1}\dfrac{\sqrt{5-x^3}-\sqrt[3]{x^2+7}}{x^2-1}$.

Ta có $I=\lim\limits_{x\to 1}\dfrac{\sqrt{5-x^3}-2+2-\sqrt[3]{x^2+7}}{x^2-1}=\lim\limits_{x\to 1}\left(\dfrac{\sqrt{5-x^3}-2}{x^2-1}+\dfrac{2-\sqrt[3]{x^2+7}}{x^2-1}\right)$

$$=\lim_{x\to 1}\left[\frac{\left(\sqrt{5-x^3}-2\right)\left(\sqrt{5-x^3}+2\right)}{(x^2-1)\left(\sqrt{5-x^3}+2\right)}+\frac{\left(2-\sqrt[3]{x^2+7}\right)\left(4+2\sqrt[3]{x^2+7}+\sqrt[3]{(x^2+7)^2}\right)}{(x^2-1)\left(4+2\sqrt[3]{x^2+7}+\sqrt[3]{(x^2+7)^2}\right)}\right]$$

$$=\lim_{x\to 1}\left[\frac{5-x^3-4}{(x^2-1)\left(\sqrt{5-x^3}+2\right)}+\frac{8-(x^2+7)}{(x^2-1)\left(4+2\sqrt[3]{x^2+7}+\sqrt[3]{(x^2+7)^2}\right)}\right]$$

$$=\lim_{x\to 1}\left[\frac{1-x^3}{(x^2-1)\left(\sqrt{5-x^3}+2\right)}+\frac{1-x^2}{(x^2-1)\left(4+2\sqrt[3]{x^2+7}+\sqrt[3]{(x^2+7)^2}\right)}\right]$$

$$=\lim_{x\to 1}\left[\frac{-(x^2+x+1)}{(x+1)\left(\sqrt{5-x^3}+2\right)}+\frac{-1}{4+2\sqrt[3]{x^2+7}+\sqrt[3]{(x^2+7)^2}}\right]=-\frac{3}{8}+\frac{-1}{12}=-\frac{11}{24}.$$

18) $I=\lim\limits_{x\to 2}\dfrac{3\sqrt[3]{4x^3-24}+\sqrt{x+2}-8\sqrt{2x-3}}{4-x^2}$.

Ta có $I=\lim\limits_{x\to 2}\dfrac{3\sqrt[3]{4x^3-24}-6+\sqrt{x+2}-2+8-8\sqrt{2x-3}}{4-x^2}$

$$=\underbrace{\lim_{x\to 2}\frac{3\sqrt[3]{4x^3-24}-6}{4-x^2}}_{I_1}+\underbrace{\lim_{x\to 2}\frac{\sqrt{x+2}-2}{4-x^2}}_{I_2}+\underbrace{\lim_{x\to 2}\frac{8-8\sqrt{2x-3}}{4-x^2}}_{I_3}$$

$$I_1=\lim_{x\to 2}\frac{3\sqrt[3]{4x^3-24}-6}{4-x^2}=\lim_{x\to 2}\frac{3\left(\sqrt[3]{4x^3-24}-2\right)\left(\sqrt[3]{4x^3-24}^2+\sqrt[3]{4x^3-24}.2+2^2\right)}{(4-x^2)\left(\sqrt[3]{4x^3-24}^2+\sqrt[3]{4x^3-24}.2+2^2\right)}$$

$$=\lim_{x\to 2}\frac{3\left(4x^3-24-8\right)}{\left(4-x^2\right)\left(\left(\sqrt[3]{4x^3-24}\right)^2+\sqrt[3]{4x^3-24}.2+2^2\right)}$$

$$=\lim_{x\to 2}\frac{3.4\left(x^3-8\right)}{\left(4-x^2\right)\left(\left(\sqrt[3]{4x^3-24}\right)^2+\sqrt[3]{4x^3-24}.2+2^2\right)}$$

$$=\lim_{x\to 2}\frac{12\left(x-2\right)\left(x^2+2x+4\right)}{\left(2-x\right)\left(2+x\right)\left(\left(\sqrt[3]{4x^3-24}\right)^2+\sqrt[3]{4x^3-24}.2+2^2\right)}$$

$$=\lim_{x\to 2}\frac{-12\left(x^2+2x+4\right)}{\left(2+x\right)\left(\left(\sqrt[3]{4x^3-24}\right)^2+\sqrt[3]{4x^3-24}.2+2^2\right)}=-\frac{144}{48}=-3.$$

$$I_2=\lim_{x\to 2}\frac{\sqrt{x+2}-2}{4-x^2}=\lim_{x\to 2}\frac{\left(\sqrt{x+2}-2\right)\left(\sqrt{x+2}+2\right)}{\left(4-x^2\right)\left(\sqrt{x+2}+2\right)}=\lim_{x\to 2}\frac{\left(x+2-4\right)}{\left(2-x\right)\left(2+x\right)\left(\sqrt{x+2}+2\right)}$$

$$=\lim_{x\to\infty}\frac{-1}{\left(2+x\right)\left(\sqrt{x+2}+2\right)}=-\frac{1}{16}$$

$$I_3=\lim_{x\to 2}\frac{8-8\sqrt{2x-3}}{4-x^2}=\lim_{x\to 2}\frac{8\left(1-\sqrt{2x-3}\right)\left(1+\sqrt{2x-3}\right)}{\left(4-x^2\right)\left(1+\sqrt{2x-3}\right)}=\lim_{x\to 2}\frac{8\left(1-\left(2x-3\right)\right)}{\left(2+x\right)\left(2-x\right)}$$

$$=\lim_{x\to 2}\frac{8.2\left(2-x\right)}{\left(2-x\right)\left(2+x\right)\left(1+\sqrt{2x+3}\right)}=\lim_{x\to 2}\frac{16}{\left(2+x\right)\left(1+\sqrt{1+2x-3}\right)}=\frac{16}{8}=2$$

$$I=-3-\frac{1}{16}+2=-\frac{17}{16}.$$

Bài 5. Tính các giới hạn sau:

1 $\lim_{x\to+\infty}\left(2x^3-3x\right)$ ĐS: $+\infty$

2. $\lim_{x\to-\infty}\left(x^3-3x^2+2\right)$ ĐS: $-\infty$

3. $\lim_{x\to+\infty}\left(-x^3-6x^2+9x+1\right)$ ĐS: $-\infty$

4. $\lim_{x\to-\infty}\left(-x^3+3x-1\right)$ ĐS: $+\infty$

5. $\lim_{x\to+\infty}\left(x^4-2x^2+1\right)$ ĐS: $+\infty$

6. $\lim_{x\to-\infty}\left(x^4-8x^2+10\right)$ ĐS: $+\infty$

7. $\lim_{x\to+\infty}\left(-x^4+2x^2+3\right)$ ĐS: $-\infty$

8. $\lim_{x\to-\infty}\left(-x^4-x^2+6\right)$ ĐS: $-\infty$

9. $\lim_{x\to\pm\infty}\sqrt{x^2-3x+4}$ ĐS: $+\infty$

10. $\lim_{x\to-\infty}\left(\sqrt{2x^2+1}+x\right)$ ĐS: $+\infty$

11. $\lim_{x\to-\infty}\left(\sqrt{x^2+x+1}+2x\right)$ ĐS: $-\infty$

12. $\lim_{x\to+\infty}\left(\sqrt{4x^2+x+1}-x\right)$ ĐS: $+\infty$

13. $\lim_{x\to+\infty}\left(\sqrt{x+1}-\sqrt{9x+1}\right)$ ĐS: $-\infty$

14. $\lim\limits_{x\to-\infty}\left(\sqrt{16x+7}+\sqrt{9x+3}\right)$ ĐS: không tồn tại giới hạn

Lời giải

1. $I=\lim\limits_{x\to+\infty}\left(2x^3-3x\right)$

Ta có $I=\lim\limits_{x\to+\infty}\left(2x^3-3x\right)=\lim\limits_{x\to+\infty}x^3\left(2-\dfrac{3}{x^2}\right)=+\infty$. (vì $\lim\limits_{x\to+\infty}x^3=+\infty$ và $\lim\limits_{x\to+\infty}\left(2-\dfrac{3}{x^2}\right)=2>0$)

2. $I=\lim\limits_{x\to-\infty}\left(x^3-3x^2+2\right)$.

Ta có $I=\lim\limits_{x\to-\infty}\left(x^3-3x^2+2\right)=\lim\limits_{x\to-\infty}x^3\left(1-\dfrac{3}{x}+\dfrac{2}{x^3}\right)=-\infty$. (vì $\lim\limits_{x\to-\infty}x^3=-\infty$ và

$\lim\limits_{x\to-\infty}\left(1-\dfrac{3}{x}+\dfrac{2}{x^3}\right)=1$).

3. $I=\lim\limits_{x\to+\infty}\left(-x^3-6x^2+9x+1\right)$.

Ta có $I=\lim\limits_{x\to+\infty}x^3\left(-1-\dfrac{6}{x}+\dfrac{9}{x^2}+\dfrac{1}{x^3}\right)=-\infty$.

(vì $\lim\limits_{x\to+\infty}x^3=+\infty$ và $\lim\limits_{x\to+\infty}\left(-1-\dfrac{6}{x}+\dfrac{9}{x^2}+\dfrac{1}{x^3}\right)=-1<0$).

4. $I=\lim\limits_{x\to-\infty}\left(-x^3+3x-1\right)$

Ta có $I=\lim\limits_{x\to-\infty}x^3\left(-1+\dfrac{3}{x^2}-\dfrac{1}{x^3}\right)=+\infty$. (vì $\lim\limits_{x\to-\infty}x^3=-\infty$ và $\lim\limits_{x\to-\infty}\left(-1+\dfrac{3}{x^2}-\dfrac{1}{x^3}\right)=-1<0$).

5. $I=\lim\limits_{x\to+\infty}\left(x^4-2x^2+1\right)$

Ta có $I=\lim\limits_{x\to+\infty}x^4\left(1-\dfrac{2}{x^2}+\dfrac{1}{x^4}\right)=+\infty$. (vì $\lim\limits_{x\to+\infty}x^4=+\infty$ và $\lim\limits_{x\to+\infty}\left(1-\dfrac{2}{x^2}+\dfrac{1}{x^4}\right)=1>0$).

6. $I=\lim\limits_{x\to-\infty}\left(x^4-8x^2+10\right)$

Ta có $I=\lim\limits_{x\to-\infty}x^4\left(1-\dfrac{8}{x^2}+\dfrac{10}{x^4}\right)=1>0$ (vì $\lim\limits_{x\to-\infty}x^4=+\infty$ và $\lim\limits_{x\to-\infty}\left(1-\dfrac{8}{x^2}+\dfrac{10}{x^4}\right)=1>0$)

7. $I=\lim\limits_{x\to+\infty}\left(-x^4+2x^2+3\right)$

Ta có $I=\lim\limits_{x\to+\infty}x^4\left(-1+\dfrac{2}{x^2}+\dfrac{3}{x^4}\right)=-\infty$. (vì $\lim\limits_{x\to+\infty}x^4=+\infty$ và $\lim\limits_{x\to+\infty}\left(-1+\dfrac{2}{x^2}+\dfrac{3}{x^4}\right)=-1<0$).

8. $I=\lim\limits_{x\to-\infty}\left(-x^4-x^2+6\right)$

Ta có $I=\lim\limits_{x\to-\infty}x^4\left(-1-\dfrac{1}{x^2}+\dfrac{6}{x^4}\right)=-\infty$. (vì $\lim\limits_{x\to-\infty}x^4=+\infty$ và $\lim\limits_{x\to-\infty}\left(-1-\dfrac{1}{x^2}+\dfrac{6}{x^4}\right)=-1$)

9. $I=\lim\limits_{x\to\pm\infty}\sqrt{x^2-3x+4}$.

Ta có $I=\lim\limits_{x\to\pm\infty}\sqrt{x^2\left(1-\dfrac{3}{x}+\dfrac{4}{x^2}\right)}=\lim\limits_{x\to\pm\infty}|x|\sqrt{\left(1-\dfrac{3}{x}+\dfrac{4}{x^2}\right)}=+\infty$.

(vì $\lim\limits_{x\to\pm\infty}|x|=+\infty$ và $\lim\limits_{x\to\pm\infty}\sqrt{\left(1-\dfrac{3}{x}+\dfrac{4}{x^2}\right)}=1>0$).

10. $I=\lim\limits_{x\to-\infty}\left(\sqrt{2x^2+1}+x\right)$

Ta có $I=\lim\limits_{x\to-\infty}\left(\sqrt{2x^2+1}+x\right)=\lim\limits_{x\to-\infty}x\left(-\sqrt{2+\frac{1}{x^2}}+1\right)=+\infty$.

(vì $\lim\limits_{x\to-\infty}x=-\infty$ và $\lim\limits_{x\to-\infty}\left(-\sqrt{2+\frac{1}{x^2}}+1\right)=-\sqrt{2}+1<0$).

11. $I=\lim\limits_{x\to-\infty}\left(\sqrt{x^2+x+1}+2x\right)=\lim\limits_{x\to-\infty}x\left(-\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}+2\right)=-\infty$.

(vì $\lim\limits_{x\to-\infty}x=-\infty$, $\lim\limits_{x\to-\infty}\left(-\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}+2\right)=1>0$).

12. $I=\lim\limits_{x\to+\infty}\left(\sqrt{4x^2+x+1}-x\right)=\lim\limits_{x\to+\infty}x\left(\sqrt{4+\frac{1}{x}+\frac{1}{x^2}}-1\right)=+\infty$

(vì $\lim\limits_{x\to+\infty}x=+\infty$, $=\lim\limits_{x\to+\infty}\left(\sqrt{4+\frac{1}{x}+\frac{1}{x^2}}-1\right)=1>0$).

13. $I=\lim\limits_{x\to+\infty}\left(\sqrt{x+1}-\sqrt{9x+1}\right)=\lim\limits_{x\to+\infty}\sqrt{x}\left(\sqrt{1+\frac{1}{x}}-\sqrt{9+\frac{1}{x}}\right)=-\infty$.

(vì $\lim\limits_{x\to+\infty}\sqrt{x}=+\infty$, $=\lim\limits_{x\to+\infty}\sqrt{x}\left(\sqrt{1+\frac{1}{x}}-\sqrt{9+\frac{1}{x}}\right)=-\infty$).

14. $I=\lim\limits_{x\to-\infty}\left(\sqrt{16x+7}+\sqrt{9x+3}\right)$

Tập xác định của hàm số $f(x)=\sqrt{16x+7}+\sqrt{9x+3}$ là $D=\left[-\frac{1}{3};+\infty\right)$.

Ta có khi $x\to-\infty$ hàm số $f(x)=\sqrt{16x+7}+\sqrt{9x+3}$ không xác định. Do đó $\lim\limits_{x\to-\infty}\left(\sqrt{16x+7}+\sqrt{9x+3}\right)$ không tồn tại.

Bài 6. **Tính các giới hạn sau:**

1. $\lim\limits_{x\to+\infty}\frac{x+2}{x-1}$. ĐS: 1

2. $\lim\limits_{x\to-\infty}\frac{2x}{x+1}$. ĐS: 2

3. $\lim\limits_{x\to+\infty}\frac{1-x}{2x-1}$. ĐS: $-\frac{1}{2}$

4. $\lim\limits_{x\to-\infty}\frac{3x-2}{x+1}$. ĐS: 3

5. $\lim\limits_{x\to+\infty}\frac{2x^3+3x-4}{-x^3-x^2+1}$. ĐS: -2

6. $\lim\limits_{x\to+\infty}\frac{3x\left(2x^2-1\right)}{(5x-1)\left(x^2+2x\right)}$. ĐS: $\frac{6}{5}$

7. $\lim\limits_{x\to-\infty}\frac{2x^4+7x^3-15}{x^4+1}$. ĐS: 2

8. $\lim\limits_{x\to+\infty}\frac{\left(4x^2+1\right)(7x-1)}{\left(2x^3-1\right)(x+3)}$. ĐS: 0

9. $\lim\limits_{x\to-\infty}\frac{(x-1)^2(5x+2)^2}{(3x+1)^4}$. ĐS: $\frac{25}{81}$

10. $\lim\limits_{x\to-\infty}\frac{(x+1)^4(1-2x)^3}{(2x+2)^5\left(x^2+3\right)}$. ĐS: $-\frac{1}{4}$

11. $\lim\limits_{x\to-\infty}\frac{\left(x^2+2\right)^2(x+2)}{\left(2x^2+1\right)(1-x)^2}$. ĐS: $-\infty$

12. $\lim\limits_{x\to-\infty}\frac{(x+2)^3(1-x)^4}{(1-2x)^5x^2}$. ĐS: $-\frac{1}{32}$

13. $\lim\limits_{x\to-\infty}\left(\dfrac{x^3}{3x^2-4}-\dfrac{x^2}{3x+2}\right)$. ĐS: $\dfrac{2}{9}$

14. $\lim\limits_{x\to-\infty}\dfrac{3x^2-x+7}{2x^3-1}$.ĐS: 0

15. $\lim\limits_{x\to+\infty}\dfrac{x^3+2x+2}{2x^4+x+3}$ ĐS: 0

16. $\lim\limits_{x\to+\infty}\dfrac{(4x^2+1)(7x-1)}{(2x^3-1)(x+3)}$ ĐS: 0

17. $\lim\limits_{x\to-\infty}\dfrac{(4x^2+1)(2x+3)}{x^2-6x+1}$ ĐS: $-\infty$

18. $\lim\limits_{x\to+\infty}\dfrac{x^3+2x+2}{2x^2+x+3}$ ĐS: $+\infty$

19. $\lim\limits_{x\to-\infty}\dfrac{x^4+2x^3+x+2}{2x^3+x+3}$ ĐS: $-\infty$

20. $\lim\limits_{x\to+\infty}\dfrac{x^4+2x^3+x+2}{2x^2-x^3}$ ĐS: $-\infty$

21. $\lim\limits_{x\to+\infty}\dfrac{x^4-x^3+11}{2x-7}$ ĐS: $+\infty$

22. $\lim\limits_{x\to+\infty}\dfrac{\sqrt{2x^4+x^2-1}}{1-2x}$ ĐS: $+\infty$

23. $\lim\limits_{x\to+\infty}\dfrac{\sqrt{x^4-x}}{1-2x}$ ĐS: $+\infty$

24. $\lim\limits_{x\to+\infty}\sqrt[3]{\dfrac{2x^5+x^3-1}{(2x^2-1)(x^3+x)}}$ ĐS: 1

25. $\lim\limits_{x\to+\infty}\dfrac{\sqrt[3]{x^3+x+1}}{2x+1}$ ĐS: 1

26. $\lim\limits_{x\to+\infty}\dfrac{\sqrt{2x^4+x^2-1}}{1-2x}$ ĐS: $-\infty$

Lời giải

1. $I=\lim\limits_{x\to+\infty}\dfrac{x+2}{x-1}=\lim\limits_{x\to+\infty}\dfrac{x\left(1+\frac{2}{x}\right)}{x\left(1-\frac{1}{x}\right)}=\lim\limits_{x\to+\infty}\dfrac{\left(1+\frac{2}{x}\right)}{\left(1-\frac{1}{x}\right)}=1$.

2. $I=\lim\limits_{x\to-\infty}\dfrac{2x}{x+1}\lim\limits_{x\to-\infty}\dfrac{2x}{x\left(1+\frac{1}{x}\right)}=\lim\limits_{x\to-\infty}\dfrac{2}{1+\frac{1}{x}}=2$.

3. $I=\lim\limits_{x\to+\infty}\dfrac{1-x}{2x-1}=\lim\limits_{x\to+\infty}\dfrac{x\left(\frac{1}{x}-1\right)}{x\left(2-\frac{1}{x}\right)}=\lim\limits_{x\to+\infty}\dfrac{\frac{1}{x}-1}{2-\frac{1}{x}}=-\dfrac{1}{2}$.

4. $I=\lim\limits_{x\to-\infty}\dfrac{3x-2}{x+1}=\lim\limits_{x\to-\infty}\dfrac{x\left(3-\frac{2}{x}\right)}{x\left(1+\frac{1}{x}\right)}=\lim\limits_{x\to-\infty}\dfrac{\left(3-\frac{2}{x}\right)}{\left(1+\frac{1}{x}\right)}=3$.

5. $I=\lim\limits_{x\to+\infty}\dfrac{2x^3+3x-4}{-x^3-x^2+1}=\lim\limits_{x\to+\infty}\dfrac{x^3\left(2+\frac{3}{x^2}-\frac{4}{x^3}\right)}{x^3\left(-1-\frac{1}{x}+\frac{1}{x^3}\right)}\lim\limits_{x\to+\infty}\dfrac{\left(2+\frac{3}{x^2}-\frac{4}{x^3}\right)}{\left(-1-\frac{1}{x}+\frac{1}{x^3}\right)}=-2$.

6. $I=\lim\limits_{x\to+\infty}\dfrac{3x.x^2\left(2-\frac{1}{x^2}\right)}{x\left(5-\frac{1}{x}\right)x^2\left(1+\frac{2}{x}\right)}=\lim\limits_{x\to+\infty}\dfrac{3\left(2-\frac{1}{x^2}\right)}{\left(5-\frac{1}{x}\right)\left(1+\frac{2}{x}\right)}=\dfrac{6}{5}$.

7. $I=\lim\limits_{x\to-\infty}\dfrac{2x^4+7x^3-15}{x^4+1}=\lim\limits_{x\to-\infty}\dfrac{x^4\left(2+\dfrac{7}{x}-\dfrac{15}{x^4}\right)}{x^4\left(1+\dfrac{1}{x^4}\right)}=\lim\limits_{x\to-\infty}\dfrac{\left(2+\dfrac{7}{x}-\dfrac{15}{x^4}\right)}{\left(1+\dfrac{1}{x^4}\right)}=2$.

8. $I=\lim\limits_{x\to+\infty}\dfrac{\left(4x^2+1\right)\left(7x-1\right)}{\left(2x^3-1\right)\left(x+3\right)}=\lim\limits_{x\to+\infty}\dfrac{x^2\left(4+\dfrac{1}{x^2}\right)x\left(7-\dfrac{1}{x}\right)}{x^3\left(2-\dfrac{1}{x^3}\right)x\left(1+\dfrac{3}{x}\right)}=\lim\limits_{x\to+\infty}\dfrac{\left(4+\dfrac{1}{x^2}\right)x\left(7-\dfrac{1}{x}\right)}{\left(2-\dfrac{1}{x^3}\right)x\left(1+\dfrac{3}{x}\right)}=0$.

9. $I=\lim\limits_{x\to-\infty}\dfrac{(x-1)^2(5x+2)^2}{(3x+1)^4}=\lim\limits_{x\to-\infty}\dfrac{x^2\left(1-\dfrac{1}{x}\right)^2x^2\left(5+\dfrac{2}{x}\right)^2}{x^4\left(3+\dfrac{1}{x}\right)^4}=\lim\limits_{x\to-\infty}\dfrac{\left(1-\dfrac{1}{x}\right)^2\left(5+\dfrac{2}{x}\right)^2}{\left(3+\dfrac{1}{x}\right)^4}=\dfrac{25}{81}$.

10. $I=\lim\limits_{x\to-\infty}\dfrac{(x+1)^4(1-2x)^3}{(2x+2)^5\left(x^2+3\right)}=\lim\limits_{x\to-\infty}\dfrac{x^4\left(1+\dfrac{1}{x}\right)^4x^3\left(\dfrac{1}{x}-2\right)^3}{x^5\left(2+\dfrac{2}{x}\right)^5x^2\left(1+\dfrac{3}{x^2}\right)}=\lim\limits_{x\to-\infty}\dfrac{\left(1+\dfrac{1}{x}\right)^4\left(\dfrac{1}{x}-2\right)^3}{\left(2+\dfrac{2}{x}\right)^5\left(1+\dfrac{3}{x^2}\right)}=-\dfrac{1}{4}$.

11. $I=\lim\limits_{x\to-\infty}\dfrac{\left(x^2+2\right)^2(x+2)}{\left(2x^2+1\right)(1-x)^2}=\lim\limits_{x\to-\infty}\dfrac{x^4\left(1+\dfrac{2}{x^2}\right)^2x\left(1+\dfrac{2}{x}\right)}{x^2\left(2+\dfrac{1}{x^2}\right)x^2\left(\dfrac{1}{x}-1\right)^2}=\lim\limits_{x\to-\infty}\dfrac{x.\left(1+\dfrac{2}{x^2}\right)^2\left(1+\dfrac{2}{x}\right)}{\left(2+\dfrac{1}{x^2}\right)\left(\dfrac{1}{x}-1\right)^2}=-\infty$

(vì $\lim\limits_{x\to-\infty}x=-\infty$, $\lim\limits_{x\to-\infty}\dfrac{\left(1+\dfrac{2}{x^2}\right)^2\left(1+\dfrac{2}{x}\right)}{\left(2+\dfrac{1}{x^2}\right)\left(\dfrac{1}{x}-1\right)^2}=\dfrac{1}{2}>0$).

12. $I=\lim\limits_{x\to-\infty}\dfrac{(x+2)^3(1-x)^4}{(1-2x)^5x^2}=\lim\limits_{x\to-\infty}\dfrac{x^3\left(1+\dfrac{2}{x}\right)^3x^4\left(\dfrac{1}{x}-1\right)^4}{x^5\left(\dfrac{1}{x}-2\right)^5.x^2}=\lim\limits_{x\to-\infty}\dfrac{\left(1+\dfrac{2}{x}\right)^3\left(\dfrac{1}{x}-1\right)^4}{\left(\dfrac{1}{x}-2\right)^5}=-\dfrac{1}{32}$.

13. $I=\lim\limits_{x\to-\infty}\left(\dfrac{x^3}{3x^2-4}-\dfrac{x^2}{3x+2}\right)$.

Ta có $I=\lim\limits_{x\to-\infty}\left(\dfrac{x^3}{3x^2-4}-\dfrac{x^2}{3x+2}\right)=\lim\limits_{x\to-\infty}\dfrac{x^3(3x+2)-x^2\left(3x^2-4\right)}{\left(3x^2-4\right)(3x+2)}=\lim\limits_{x\to-\infty}\dfrac{2x^3+4x^2}{\left(3x^2-4\right)(3x+2)}$

$=\lim\limits_{x\to-\infty}\dfrac{x^3\left(2+\dfrac{4}{x}\right)}{x^2\left(3-\dfrac{4}{x}\right)x\left(3+\dfrac{2}{x}\right)}=\lim\limits_{x\to-\infty}\dfrac{\left(2+\dfrac{4}{x}\right)}{\left(3-\dfrac{4}{x}\right)\left(3+\dfrac{2}{x}\right)}=\dfrac{2}{9}$.

14. $I=\lim\limits_{x\to-\infty}\dfrac{3x^2-x+7}{2x^3-1}=\lim\limits_{x\to-\infty}\dfrac{x^2\left(\dfrac{3}{x}-\dfrac{1}{x}+\dfrac{7}{x^2}\right)}{x^3\left(2-\dfrac{1}{x^3}\right)}\ \lim\limits_{x\to-\infty}\dfrac{\left(\dfrac{3}{x}-\dfrac{1}{x}+\dfrac{7}{x^2}\right)}{\left(2-\dfrac{1}{x^3}\right)}=0$.

Bài 7. Tính các giới hạn sau:

1. $\lim\limits_{x\to 1^+}\dfrac{2x-3}{x-1}$. ĐS: $-\infty$

2. $\lim\limits_{x\to 2^+}\dfrac{x-15}{x-2}$. ĐS: $-\infty$

3. $\lim\limits_{x\to 3^-}\dfrac{2-x}{3-x}$. ĐS: $+\infty$

4. $\lim\limits_{x\to 4^-}\dfrac{x-5}{(x-4)^2}$. ĐS: $-\infty$

5. $\lim\limits_{x\to 2^-}\dfrac{-3x+1}{x-2}$. ĐS: $+\infty$

6. $\lim\limits_{x\to 1^-}\dfrac{3x-1}{x-1}$. ĐS: $-\infty$

7. $\lim\limits_{x\to 2^+}\dfrac{6-5x}{4x-8}$. ĐS: $-\infty$

8. $\lim\limits_{x\to 2^+}\dfrac{x+1}{2x-4}$. ĐS: $+\infty$

9. $\lim\limits_{x\to 3^+}\dfrac{|x-3|}{5x-15}$. ĐS: $\dfrac{1}{5}$

10. $\lim\limits_{x\to (-3)^-}\dfrac{7x-1}{|x+3|}$.ĐS: $-\infty$

11. $\lim\limits_{x\to 2^-}\dfrac{|2-x|}{2x^2-5x+2}$. ĐS: $\dfrac{1}{3}$

12. $\lim\limits_{x\to 1^+}\dfrac{|x-1|}{2x^3+x-3}$. ĐS: $\dfrac{1}{7}$

13. $\lim\limits_{x\to 1^-}\dfrac{|x-1|}{2x^3+x-3}$. ĐS: $-\dfrac{1}{7}$

14. $\lim\limits_{x\to 2^+}\dfrac{|x^2-3x+2|}{x-2}$.ĐS: 1

15. $\lim\limits_{x\to 3}\dfrac{|x^2-9|}{x-3}$ ĐS: không tồn tại

16. $\lim\limits_{x\to 4}\dfrac{x-4}{|x^2+x-20|}$ ĐS: không tồn tại

17. $\lim\limits_{x\to 2^-}\dfrac{|x-2|}{\sqrt{x-1}-1}$ ĐS: 2

18. $\lim\limits_{x\to 3^-}\dfrac{|x-3|}{\sqrt{5x-11}-2}$ ĐS: $-\dfrac{4}{5}$

19. $\lim\limits_{x\to 2^-}\dfrac{|x-2|}{\sqrt[3]{x-1}-1}$ ĐS: -3

20. $\lim\limits_{x\to 5^-}\dfrac{|x^2-25|}{\sqrt[3]{x-4}-1}$ ĐS: -30

21. $\lim\limits_{x\to 3^+}\dfrac{3x-8}{(3-x)^2}$ ĐS: $+\infty$

22. $\lim\limits_{x\to 2}\dfrac{\sqrt[3]{x+25}-3}{x^2-x-2}$ ĐS: $\dfrac{1}{81}$

23. $\lim\limits_{x\to 2^+}\dfrac{3x+2}{\sqrt{4x^2-16}}$ ĐS: $+\infty$

24. $\lim\limits_{x\to 0^+}\dfrac{x+\sqrt{x}}{x-\sqrt{x}}$ ĐS: -1

25. $\lim\limits_{x\to 2^+}\dfrac{4-x^2}{\sqrt{2-x}}$ ĐS: 0

26. $\lim\limits_{x\to 0^+}\dfrac{x+2\sqrt{x}}{x-\sqrt{x}}$ ĐS: -2

27. $\lim\limits_{x\to (-1)^+}\dfrac{\sqrt{(x+2)(x+1)}}{\sqrt{x+1}-x-1}$ ĐS: 1

28. $\lim\limits_{x\to 3^-}\dfrac{\sqrt{x^2-6x+9}}{x^2-9}$ ĐS: $-\dfrac{1}{6}$

29. $\lim\limits_{x\to 1^-}\dfrac{\sqrt{x^2-4x+3}}{-x^2+6x-5}$ ĐS: $-\infty$

30. $\lim\limits_{x\to (-1)^+}\dfrac{x^2+3x+2}{\sqrt{x^5+x^4}}$ ĐS: 0

31. $\lim\limits_{x\to 2^+}(x-2)\sqrt{\dfrac{x}{x^2-4}}$ ĐS: 0

32. $\lim\limits_{x\to (-1)^+}(x^3+1)\sqrt{\dfrac{x}{x^2-1}}$ ĐS: 0

33. $\lim\limits_{x\to 1^+}(1-x)\sqrt{\dfrac{x+5}{x^2+2x-3}}$ ĐS: 0

34. $\lim\limits_{x\to 1^-}\dfrac{x\sqrt{1-x}}{2\sqrt{1-x}+1-x}$ ĐS: $\dfrac{1}{2}$

35. $\lim\limits_{x\to 0^+}\left(2x\sqrt{\dfrac{1-x}{x}}\right)$ ĐS: 0

36. $\lim\limits_{x\to (-3)^+}\dfrac{2x^2+5x-3}{(x+3)^2}$ ĐS: $-\infty$

37. $\lim\limits_{x\to 2^-}\left(\dfrac{1}{x-2}-\dfrac{1}{x^2-4}\right)$ ĐS: $-\infty$	38. $\lim\limits_{x\to 1^-}\dfrac{\sqrt{x^3-3x+2}}{x^2-5x+4}$ ĐS: $-\dfrac{\sqrt{3}}{5}$

Lời giải

1 $\lim\limits_{x\to 1^+}\dfrac{2x-3}{x-1}=-\infty$ vì $\begin{cases}\lim\limits_{x\to 1^+}(2x-3)=-1\\ \lim\limits_{x\to 1^+}(x-1)=0\\ x-1>0,\forall x\to 1^+\end{cases}$.

2. $\lim\limits_{x\to 2^+}\dfrac{x-15}{x-2}=-\infty$ vì $\begin{cases}\lim\limits_{x\to 2^+}(x-15)=-13\\ \lim\limits_{x\to 2^+}(x-2)=0\\ x-2>0,\forall x\to 2^+\end{cases}$.

3. $\lim\limits_{x\to 3^-}\dfrac{2-x}{3-x}=+\infty$ vì $\begin{cases}\lim\limits_{x\to 3^-}(2-x)=-1\\ \lim\limits_{x\to 3^-}(3-x)=0\\ 3-x>0,\forall x\to 3^-\end{cases}$.

4. $\lim\limits_{x\to 4^-}\dfrac{x-5}{(x-4)^2}=-\infty$ vì $\begin{cases}\lim\limits_{x\to 4^-}(x-5)=-1\\ \lim\limits_{x\to 4^-}(x-4)^2=0\\ (x-4)^2>0,\forall x\to 4^-\end{cases}$.

5. $\lim\limits_{x\to 2^-}\dfrac{-3x+1}{x-2}=+\infty$ vì $\begin{cases}\lim\limits_{x\to 2^-}(-3x+1)=-5\\ \lim\limits_{x\to 2^-}(x-2)=0\\ x-2<0,\forall x\to 2^-\end{cases}$.

6. $\lim\limits_{x\to 1^-}\dfrac{3x-1}{x-1}=-\infty$ vì $\begin{cases}\lim\limits_{x\to 1^-}(3x-1)=2\\ \lim\limits_{x\to 1^-}(x-1)=0\\ x-1<0,\forall x\to 1^-\end{cases}$.

7. $\lim\limits_{x\to 2^+}\dfrac{6-5x}{4x-8}=-\infty$ vì $\begin{cases}\lim\limits_{x\to 2^+}(6-5x)=-4\\ \lim\limits_{x\to 2^+}(4x-8)=0\\ 4x-8>0,\forall x\to 2^+\end{cases}$.

8. $\lim\limits_{x\to 2^+}\dfrac{x+1}{2x-4}=+\infty$ vì $\begin{cases}\lim\limits_{x\to 2^+}(x+1)=3\\ \lim\limits_{x\to 2^+}(2x-4)=0\\ 2x-4>0,\forall x\to 2^+\end{cases}$.

9. Do $x\to 3^+$ nên $|x-3|=x-3$ suy ra $\lim\limits_{x\to 3^+}\dfrac{|x-3|}{5x-15}=\lim\limits_{x\to 3^+}\dfrac{x-3}{5x-15}=\lim\limits_{x\to 3^+}\dfrac{1}{5}=\dfrac{1}{5}$.

10. $\lim\limits_{x\to(-3)^-}\dfrac{7x-1}{|x+3|}=-\infty$ vì $\begin{cases}\lim\limits_{x\to(-3)^-}(7x-1)=-22\\ \lim\limits_{x\to(-3)^-}|x+3|=0\\ |x+3|>0,\forall x\to(-3)^-\end{cases}$.

11. Do $x\to 2^-$ nên $|2-x|=2-x$ suy ra $\lim\limits_{x\to 2^-}\dfrac{|2-x|}{2x^2-5x+2}=\lim\limits_{x\to 2^-}\dfrac{2-x}{(2x-1)(x-2)}$

$=\lim\limits_{x\to 2^-}\dfrac{1}{2x-1}=\dfrac{1}{3}$.

12. Do $x\to 1^+$ nên $|x-1|=x-1$ suy ra $\lim\limits_{x\to 1^+}\dfrac{|x-1|}{2x^3+x-3}=\lim\limits_{x\to 1^+}\dfrac{x-1}{(x-1)(2x^2+2x+3)}$

$\lim\limits_{x\to 1^+}\dfrac{1}{2x^2+2x+3}=\dfrac{1}{7}$.

13. Do $x\to 1^-$ nên $|x-1|=1-x$ suy ra $\lim\limits_{x\to 1^-}\dfrac{|x-1|}{2x^3+x-3}=\lim\limits_{x\to 1^-}\dfrac{1-x}{(x-1)(2x^2+2x+3)}$

$\lim\limits_{x\to 1^-}\dfrac{-1}{2x^2+2x+3}=-\dfrac{1}{7}$.

14. Ta có $x^2-3x+2=(x-1)(x-2)$, do $x\to 2^+$ nên $x^2-3x+2>0$, suy ra

$\lim\limits_{x\to 2^+}\dfrac{|x^2-3x+2|}{x-2}=\lim\limits_{x\to 2^+}\dfrac{(x-1)(x-2)}{x-2}=\lim\limits_{x\to 2^+}(x-1)=1$.

15. Ta có $\lim\limits_{x\to 3}\dfrac{|x^2-9|}{x-3}=\lim\limits_{x\to 3}\dfrac{|(x+3)(x-3)|}{x-3}$

TH1: $x>3$ ta có $\lim\limits_{x\to 3^+}\dfrac{|x^2-9|}{x-3}=\lim\limits_{x\to 3^+}\dfrac{(x+3)(x-3)}{x-3}=\lim\limits_{x\to 3^+}(x+3)=6$.

TH2: $x<3$ ta có $\lim\limits_{x\to 3^-}\dfrac{|x^2-9|}{x-3}=\lim\limits_{x\to 3^-}\dfrac{-(x+3)(x-3)}{x-3}=\lim\limits_{x\to 3^-}(-x-3)=-6$.

Do $\lim\limits_{x\to 3^+}\dfrac{|x^2-9|}{x-3}\neq\lim\limits_{x\to 3^-}\dfrac{|x^2-9|}{x-3}$ nên không tồn tại $\lim\limits_{x\to 3}\dfrac{|x^2-9|}{x-3}$.

16. Ta có $\lim\limits_{x\to 4}\dfrac{x-4}{|x^2+x-20|}=\lim\limits_{x\to 4}\dfrac{x-4}{|(x-4)(x+5)|}$

TH1: $x>4$, ta có $\lim\limits_{x\to 4^+}\dfrac{x-4}{|x^2+x-20|}=\lim\limits_{x\to 4^+}\dfrac{x-4}{(x-4)(x+5)}=\lim\limits_{x\to 4^+}\dfrac{1}{x+5}=\dfrac{1}{9}$

TH2: $x<4$, ta có $\lim\limits_{x\to 4^+}\dfrac{x-4}{|x^2+x-20|}=\lim\limits_{x\to 4^-}\dfrac{x-4}{(4-x)(x+5)}=\lim\limits_{x\to 4^-}\dfrac{-1}{x+5}=\dfrac{-1}{9}$

Do $\lim\limits_{x\to 4^+}\dfrac{x-4}{|x^2+x-20|}\neq\lim\limits_{x\to 4^-}\dfrac{x-4}{|x^2+x-20|}$ nên không tồn tại $\lim\limits_{x\to 4}\dfrac{x-4}{|x^2+x-20|}$.

17. Do $x\to 2^-$ nên $|x-2|=2-x$ suy ra $\lim\limits_{x\to 2^-}\dfrac{|x-2|}{\sqrt{x-1}-1}=\lim\limits_{x\to 2^-}\dfrac{(x-2)(\sqrt{x-1}+1)}{x-1-1}$

$\lim\limits_{x\to 2^-}(\sqrt{x-1}+1)=2$.

18. Do $x\to 3^-$ nên $|x-2|=3-x$ suy ra $\lim\limits_{x\to 3^-}\dfrac{|x-3|}{\sqrt{5x-11}-2}=\lim\limits_{x\to 3^-}\dfrac{(3-x)(\sqrt{5x-11}+2)}{5x-11-4}$

$=\lim\limits_{x\to 3^-}\dfrac{-(\sqrt{5x-11}+2)}{5}=-\dfrac{4}{5}$.

19. Do $x \to 2^-$ nên $|x-2| = 2-x$ suy ra $\lim\limits_{x\to 2^-}\dfrac{|x-2|}{\sqrt[3]{x-1}-1} = \lim\limits_{x\to 2^-}\dfrac{(2-x)\left(\sqrt[3]{x-1}^2+\sqrt[3]{x-1}+1\right)}{x-1-1}$

$= \lim\limits_{x\to 2^-}\left(-\left(\sqrt[3]{x-1}^2+\sqrt[3]{x-1}+1\right)\right) = -3$.

20. Ta có $x^2-25=(x-5)(x+5)$, do $x\to 5^-$ nên $x^2-25<0$, suy ra $\lim\limits_{x\to 5^-}\dfrac{|x^2-25|}{\sqrt[3]{x-4}-1}$

$= \lim\limits_{x\to 5^-}\dfrac{(25-x^2)\left(\sqrt[3]{x-4}^2+\sqrt[3]{x-4}+1\right)}{x-4-1} = \lim\limits_{x\to 5^-}\left(-(5+x)\right)\left(\sqrt[3]{x-4}^2+\sqrt[3]{x-4}+1\right) = -30$.

21. $\lim\limits_{x\to 3^+}\dfrac{3x-8}{(3-x)^2} = +\infty$, vì $\begin{cases}\lim\limits_{x\to 3^+}(3x-8)=1\\ \lim\limits_{x\to 3^+}(3-x)^2=0\\ (3-x)^2>0, \forall x\to 3^+\end{cases}$.

22. Ta có $\lim\limits_{x\to 2}\dfrac{\sqrt[3]{x+25}-3}{x^2-x-2} = \lim\limits_{x\to 2}\dfrac{x+25-27}{(x-2)(x+1)\left(\sqrt[3]{x+25}^2+3\sqrt[3]{x+25}+9\right)}$

$= \lim\limits_{x\to 2}\dfrac{1}{(x+1)\left(\sqrt[3]{x+25}^2+3\sqrt[3]{x+25}+9\right)} = \dfrac{1}{81}$.

23. $\lim\limits_{x\to 2^+}\dfrac{3x+2}{\sqrt{4x^2-16}} = +\infty$, vì $\begin{cases}\lim\limits_{x\to 2^+}(3x+2)=8\\ \lim\limits_{x\to 2^+}\sqrt{4x^2-16}=0\\ \sqrt{4x^2-16}>0, \forall x\to 2^+\end{cases}$.

24. $\lim\limits_{x\to 0^+}\dfrac{x+\sqrt{x}}{x-\sqrt{x}} = \lim\limits_{x\to 0^+}\dfrac{\sqrt{x}(\sqrt{x}+1)}{\sqrt{x}(\sqrt{x}-1)} = \lim\limits_{x\to 0^+}\dfrac{\sqrt{x}+1}{\sqrt{x}-1} = -1$.

25. $\lim\limits_{x\to 2^+}\dfrac{4-x^2}{\sqrt{2-x}} = \lim\limits_{x\to 2^+}\dfrac{(2-x)(2+x)}{\sqrt{2-x}} = \lim\limits_{x\to 2^+}\sqrt{2-x}(2+x) = 0$.

26. $\lim\limits_{x\to 0^+}\dfrac{x+2\sqrt{x}}{x-\sqrt{x}} = \lim\limits_{x\to 0^+}\dfrac{\sqrt{x}(\sqrt{x}+2)}{\sqrt{x}(\sqrt{x}-1)} = \lim\limits_{x\to 0^+}\dfrac{\sqrt{x}+2}{\sqrt{x}-1} = -2$.

27. Ta có $\lim\limits_{x\to(-1)^+}\dfrac{\sqrt{(x+2)(x+1)}}{\sqrt{x+1}-x-1} = \lim\limits_{x\to(-1)^+}\dfrac{\sqrt{x+2}\sqrt{x+1}}{\sqrt{x+1}\left(1-\sqrt{x+1}\right)} = \lim\limits_{x\to(-1)^+}\dfrac{\sqrt{x+2}}{1-\sqrt{x+1}} = 1$.

28. Ta có $\sqrt{x^2-6x+9} = \sqrt{(x-3)^2} = |x-3|$, do $x\to 3^-$ nên $\sqrt{x^2-6x+9} = 3-x$, suy ra

$$\lim\limits_{x\to 3^-}\dfrac{\sqrt{x^2-6x+9}}{x^2-9} = \lim\limits_{x\to 3^-}\dfrac{\sqrt{x^2-6x+9}}{x^2-9} = \lim\limits_{x\to 3^-}\dfrac{3-x}{(x-3)(x+3)} = \lim\limits_{x\to 3^-}\dfrac{-1}{x+3} = -\dfrac{1}{6}.$$

29. Do $x\to 1^-$ nên $x-1<0$, từ đó ta có

$\lim\limits_{x\to 1^-}\dfrac{\sqrt{x^2-4x+3}}{-x^2+6x-5} = \lim\limits_{x\to 1^-}\dfrac{\sqrt{(x-1)(x-3)}}{-(x-1)(x-5)} = \lim\limits_{x\to 1^-}\dfrac{\sqrt{1-x}\sqrt{3-x}}{-(x-1)(x-5)} = \lim\limits_{x\to 1^-}\dfrac{\sqrt{3-x}}{\sqrt{1-x}(x-5)}$

$= \lim\limits_{x\to 1^-}\left(\dfrac{1}{\sqrt{1-x}}\cdot\dfrac{\sqrt{3-x}}{x-5}\right) = -\infty$.

vì $\lim\limits_{x\to1^-}\frac{\sqrt{3-x}}{x-5}=-\frac{\sqrt{2}}{4}$ và $\lim\limits_{x\to1^-}\left(\frac{1}{\sqrt{1-x}}\right)=+\infty$.

30. $\lim\limits_{x\to(-1)^+}\frac{x^2+3x+2}{\sqrt{x^5+x^4}}=\lim\limits_{x\to(-1)^+}\frac{(x+1)(x+2)}{x^2\sqrt{x+1}}=\lim\limits_{x\to(-1)^+}\frac{\sqrt{x+1}(x+2)}{x^2}=0$.

31. $\lim\limits_{x\to2^+}(x-2)\sqrt{\frac{x}{x^2-4}}=\lim\limits_{x\to2^+}(x-2)\sqrt{\frac{x}{(x-2)(x+2)}}=\lim\limits_{x\to2^+}\sqrt{\frac{x(x-2)}{x+2}}=0$.

32. Ta có $\lim\limits_{x\to(-1)^+}(x^3+1)\sqrt{\frac{x}{x^2-1}}=\lim\limits_{x\to(-1)^+}(x+1)(x^2-x+1)\sqrt{\frac{x}{(x-1)(x+1)}}$

$=\lim\limits_{x\to(-1)^+}(x^2-x+1)\sqrt{\frac{x(x+1)}{x-1}}=0$.

33. Do $x\to1^+$ nên $1-x<0$, vì thế ta có

$\lim\limits_{x\to1^+}(1-x)\sqrt{\frac{x+5}{x^2+2x-3}}=\lim\limits_{x\to1^+}\left(\sqrt{\frac{(x+5)(x-1)^2}{(x-1)(x+3)}}\right)=\lim\limits_{x\to1^+}\left(\sqrt{\frac{(x+5)(x-1)}{x+3}}\right)=0$.

34. $\lim\limits_{x\to1^-}\frac{x\sqrt{1-x}}{2\sqrt{1-x}+1-x}=\lim\limits_{x\to1^-}\frac{x\sqrt{1-x}}{\sqrt{1-x}\left(2+\sqrt{1-x}\right)}=\lim\limits_{x\to1^-}\frac{x}{2+\sqrt{1-x}}=\frac{1}{2}$.

35. $\lim\limits_{x\to0^+}\left(2x\sqrt{\frac{1-x}{x}}\right)=\lim\limits_{x\to0^+}\left(2\sqrt{\frac{x^2(1-x)}{x}}\right)=\lim\limits_{x\to0^+}2\sqrt{x(1-x)}=0$.

36. $\lim\limits_{x\to(-3)^+}\frac{2x^2+5x-3}{(x+3)^2}=\lim\limits_{x\to(-3)^+}\frac{(2x-1)(x+3)}{(x+3)^2}=\lim\limits_{x\to(-3)^+}\frac{2x-1}{x+3}=-\infty$.

37. $\lim\limits_{x\to2^-}\left(\frac{1}{x-2}-\frac{1}{x^2-4}\right)=\lim\limits_{x\to2^-}\left(\frac{x+2-1}{(x-2)(x+2)}\right)=\lim\limits_{x\to2^-}\left(\frac{x+1}{x+2}.\frac{1}{x-2}\right)=-\infty$.

38. Do $x\to1^-$ nên $x-1<0$, suy ra $\sqrt{(x-1)^2}=|x-1|=1-x$ nên ta có

$\lim\limits_{x\to1^-}\frac{\sqrt{x^3-3x+2}}{x^2-5x+4}=\lim\limits_{x\to1^-}\frac{\sqrt{(x+2)(x-1)^2}}{(x-1)(x+4)}=\lim\limits_{x\to1^-}\frac{(1-x)\sqrt{x+2}}{(x-1)(x+4)}=\lim\limits_{x\to1^-}\frac{-\sqrt{x+2}}{x+4}=-\frac{\sqrt{3}}{5}$.

Bài 8. Tính các giới hạn sau:

1) $\lim\limits_{x\to0}\frac{\sin5x}{x}$. ĐS: 5

2) $\lim\limits_{x\to0}\frac{\tan2x}{3x}$. ĐS: $\frac{2}{3}$

3) $\lim\limits_{x\to0}\frac{1-\cos x}{x^2}$. ĐS: $\frac{1}{2}$

4) $\lim\limits_{x\to0}\frac{\sin5x.\sin3x.\sin x}{45x^3}$. ĐS: $\frac{1}{3}$

5) $\lim\limits_{x\to0}\frac{1-\cos5x}{1-\cos3x}$

6) $\lim\limits_{x\to0}\frac{1-\cos^2 2x}{x.\sin x}$. ĐS: 4

7) $\lim\limits_{x\to0}\frac{x\sin ax}{1-\cos ax}(a\neq0)$. ĐS: $\frac{2}{a}$

8) $\lim\limits_{x\to0}\frac{1-\cos ax}{1-\cos bx}$. ĐS: $\frac{a^2}{b^2}$

9) $\lim\limits_{x\to0}\frac{1-\cos x}{x^2};(a\neq0)$ ĐS: $\frac{a^2}{2}$

10) $\lim\limits_{x\to0}\frac{\sin x-\tan x}{x^3}$ ĐS: $-\frac{1}{2}$

11) $\lim\limits_{x\to 0}\dfrac{\tan x-\sin x}{\sin^3 x}$. ĐS: $\dfrac{1}{2}$

12) $\lim\limits_{x\to a}\dfrac{\sin x-\sin a}{x-a}$. ĐS: $\cos a$

13) $\lim\limits_{x\to b}\dfrac{\cos x-\cos b}{x-b}$ ĐS: $-\sin b$

14) $\lim\limits_{x\to 0}\dfrac{1-\sqrt{2x+1}}{\sin 2x}$ ĐS: $\dfrac{-1}{2}$

15) $\lim\limits_{x\to 0}\dfrac{\cos(a+x)-\cos(a-x)}{x}$ ĐS: $-2\sin a$

16) $\lim\limits_{x\to c}\dfrac{\tan x-\tan c}{x-c}$ ĐS: $\dfrac{1}{\cos^2 c}$

17) $\lim\limits_{x\to 0}\dfrac{1-\cos^3 x}{x\sin x}$ ĐS: $\dfrac{3}{2}$

18) $\lim\limits_{x\to a}\dfrac{\sin^2 x-\sin^2 a}{x^2-a^2}$ ĐS: $\dfrac{\sin 2a}{2a}$

19) $\lim\limits_{x\to 0}\dfrac{\cos\alpha x-\cos\beta x}{x^2}$ ĐS: $\dfrac{\beta^2-\alpha^2}{2}$

20) $\lim\limits_{x\to -2}\dfrac{x^3+8}{\tan(x+2)}$ ĐS:12

21) $\lim\limits_{x\to 0}\dfrac{1-\cos x\cos 2x\cos 3x}{1-\cos x}$. ĐS:14

22) $\lim\limits_{x\to 0}\dfrac{\sin(a+2x)-2\sin(a+x)+\sin a}{x^2}$. ĐS: $-\sin(\alpha)$

23) $\lim\limits_{x\to 0}\dfrac{\sin ax+\tan bx}{(a+b)x};(a+b\neq 0)$ ĐS: 1

24) $\lim\limits_{x\to 0}\dfrac{\cos 3x-\cos 5x\cos 7x}{x^2}$ ĐS: $-\dfrac{33}{2}$

25) $\lim\limits_{x\to 0}\dfrac{\cos ax-\cos bx\cos cx}{1-\cos x}$. ĐS: $\dfrac{b^2-a^2-c^2}{2}$

26) $\lim\limits_{x\to 0}\dfrac{\sin(a+x)-\sin(a-x)}{\tan(a+x)-\tan(a-x)}$ ĐS: $\cos^3 a$

27) $\lim\limits_{x\to 0}\dfrac{\sqrt{2x+1}-\sqrt[3]{x^2+1}}{\sin x}$. ĐS: 1

28) $\lim\limits_{x\to 0}\dfrac{\sin^2 2x-\sin x\sin 4x}{x^4}$ ĐS: 6

29) $\lim\limits_{x\to -\frac{\pi}{2}}\dfrac{\cos x}{x+\dfrac{\pi}{2}}$. ĐS: 1

30) $\lim\limits_{x\to 0}\dfrac{\sin x-\sin 2x}{x\left(1-\sin^2\dfrac{x}{2}\right)}$ ĐS: -1

31) $\lim\limits_{x\to 0}\dfrac{\sqrt{1+x^2}-\cos x}{x^2}$. ĐS: 1

32) $\lim\limits_{x\to 0}\dfrac{\sqrt{1+\tan x}-\sqrt{1+\sin x}}{x^3}$ ĐS: $\dfrac{1}{4}$

33) $\lim\limits_{x\to 0}\dfrac{1-\cos 5x\cos 7x}{\sin^2 11x}$. ĐS: $\dfrac{37}{121}$

34) $\lim\limits_{x\to 1}\dfrac{\sqrt{x+3}-2}{\tan(x-1)}$ ĐS: $\dfrac{1}{4}$

35) $\lim\limits_{x\to \pi}\dfrac{1+\cos x}{(x-\pi)^2}$. ĐS: $\dfrac{1}{2}$

36) $\lim\limits_{x\to 1}\dfrac{\sin(x-1)}{x^2-4x+3}$ ĐS: $-\dfrac{1}{2}$

37) $\lim\limits_{x\to 0}\dfrac{\sqrt{x^2+1}-\cos 2x}{x^2}$. ĐS: $\dfrac{5}{2}$

38) $\lim\limits_{x\to 0}\dfrac{1-\cos x\sqrt{\cos 2x}}{x^2}$ ĐS: $\dfrac{3}{2}$

Lời giải.

1) $\lim\limits_{x\to 0}\dfrac{\sin 5x}{x}=\lim\limits_{x\to 0}\left(5.\dfrac{\sin 5x}{5x}\right)=5.$

2) $\lim\limits_{x\to 0}\dfrac{\tan 2x}{3x}=\lim\limits_{x\to 0}\left(\dfrac{2}{3}\cdot\dfrac{\tan 2x}{2x}\right)=\dfrac{2}{3}.$

3) $\lim\limits_{x\to0}\frac{1-\cos x}{x^2}=\lim\limits_{x\to0}\left(\frac{2\sin^2\left(\frac{x}{2}\right)}{x^2}\right)=\lim\limits_{x\to0}\left(2\cdot\frac{1}{4}\left(\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right)^2\right)=\frac{1}{2}$.

4) $\lim\limits_{x\to0}\frac{\sin5x\sin3x\sin x}{45x^3}=\lim\limits_{x\to0}\left(\frac{1}{3}\cdot\frac{\sin5x}{5x}\cdot\frac{\sin3x}{3x}\cdot\frac{\sin x}{x}\right)=\frac{1}{3}$.

5) $\lim\limits_{x\to0}\frac{1-\cos5x}{1-\cos3x}=\lim\limits_{x\to0}\frac{2\sin^2\frac{5x}{2}}{2\sin^2\frac{3x}{2}}=\lim\limits_{x\to0}\left(\frac{25}{4}\cdot\left(\frac{\sin\frac{5x}{2}}{\frac{5x}{2}}\right)^2\cdot\frac{4}{9}\cdot\left(\frac{\frac{3x}{2}}{\sin\frac{3x}{2}}\right)^2\right)=\frac{25}{9}$.

6) $\lim\limits_{x\to0}\frac{1-\cos^2 2x}{x\sin x}=\lim\limits_{x\to0}\frac{\sin^2 2x}{x\sin x}=\lim\limits_{x\to0}\frac{4\sin x\cos^2 x}{x}=\lim\limits_{x\to0}\left(4\cos^2 x\cdot\frac{\sin x}{x}\right)=4$.

7) $\lim\limits_{x\to0}\frac{x\sin ax}{1-\cos ax}=\lim\limits_{x\to0}\frac{x\sin ax}{2\sin^2\frac{ax}{2}}=\lim\limits_{x\to0}\left(\frac{1}{2}\cdot a\cdot\frac{\sin ax}{ax}\cdot\frac{4}{a^2}\cdot\left(\frac{\frac{ax}{2}}{\sin\frac{ax}{2}}\right)^2\right)=\frac{2}{a}$.

8) $\lim\limits_{x\to0}\frac{1-\cos ax}{1-\cos bx}=\lim\limits_{x\to0}\frac{2\sin^2\frac{ax}{2}}{2\sin^2\frac{bx}{2}}=\lim\limits_{x\to0}\left(\frac{a^2}{4}\cdot\left(\frac{\sin\frac{ax}{2}}{\frac{ax}{2}}\right)^2\cdot\frac{4}{b^2}\cdot\left(\frac{\frac{bx}{2}}{\sin\frac{bx}{2}}\right)^2\right)=\frac{a^2}{b^2}$.

9) $\lim\limits_{x\to0}\frac{1-\cos ax}{x^2}=\lim\limits_{x\to0}\frac{2\sin^2\frac{ax}{2}}{x^2}=\lim\limits_{x\to0}\left(2\cdot\frac{a^2}{4}\cdot\left(\frac{\sin\frac{ax}{2}}{\frac{ax}{2}}\right)^2\right)=\frac{a^2}{2}$.

10) Ta có $\lim\limits_{x\to0}\frac{\sin x-\tan x}{x^3}=\lim\limits_{x\to0}\frac{\sin x(\cos x-1)}{x^3\cos x}$

$=\lim\limits_{x\to0}\frac{-2\sin x\sin^2\frac{x}{2}}{x^3\cos x}=\lim\limits_{x\to0}\left(-\frac{2}{\cos x}\cdot\frac{\sin x}{x}\cdot\frac{1}{4}\cdot\left(\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right)^2\right)=-\frac{1}{2}$.

11) $\lim\limits_{x\to0}\frac{\tan x-\sin x}{\sin^3 x}=\lim\limits_{x\to0}\frac{\sin x(1-\cos x)}{\cos x\sin x(1-\cos^2 x)}=\lim\limits_{x\to0}\left(\frac{1}{\cos x(1+\cos x)}\right)=\frac{1}{2}$.

12) Ta có $\lim\limits_{x\to a}\frac{\sin x-\sin a}{x-a}=\lim\limits_{x\to a}\frac{2\cos\frac{x+a}{2}\sin\frac{x-a}{2}}{x-a}=\lim\limits_{x\to a}\left(\cos\frac{x+a}{2}\cdot\frac{\sin\frac{x-a}{2}}{\frac{x-a}{2}}\right)=\cos a$.

13) Ta có $\lim\limits_{x\to b}\dfrac{\cos x-\cos b}{x-b}=\lim\limits_{x\to b}\dfrac{-2\sin\dfrac{x+b}{2}\sin\dfrac{x-b}{2}}{x-b}=\lim\limits_{x\to b}\left(-\sin\dfrac{x+b}{2}\cdot\dfrac{\sin\dfrac{x-b}{2}}{\dfrac{x-b}{2}}\right)=-\sin b.$

14) Ta có $\lim\limits_{x\to 0}\dfrac{1-\sqrt{2x+1}}{\sin 2x}=\lim\limits_{x\to 0}\dfrac{1-2x-1}{\sin 2x\left(1+\sqrt{2x+1}\right)}=\lim\limits_{x\to 0}\left(-\dfrac{1}{1+\sqrt{2x+1}}\cdot\dfrac{2x}{\sin 2x}\right)=-\dfrac{1}{2}.$

15) Ta có $\lim\limits_{x\to 0}\dfrac{\cos(a+x)-\cos(a-x)}{x}=\lim\limits_{x\to 0}\dfrac{-2\sin\dfrac{a+x+a-x}{2}\sin\dfrac{a+x-a+x}{2}}{x}$

$=\lim\limits_{x\to 0}\left(-2\sin a\cdot\dfrac{\sin x}{x}\right)=-2\sin a.$

16) $\lim\limits_{x\to c}\dfrac{\tan x-\tan c}{x-c}=\lim\limits_{x\to c}\dfrac{\sin(x-c)}{(x-c)\cos x\cos c}=\lim\limits_{x\to c}\left(\dfrac{\sin(x-c)}{x-c}\cdot\dfrac{1}{\cos x\cos c}\right)=\dfrac{1}{\cos^2 c}.$

17) Ta có $\lim\limits_{x\to 0}\dfrac{1-\cos^3 x}{x\sin x}=\lim\limits_{x\to 0}\dfrac{(1-\cos x)\left(1+\cos x+\cos^2 x\right)}{x\sin x}$

$=\lim\limits_{x\to 0}\dfrac{2\sin^2\dfrac{x}{2}\left(1+\cos x+\cos^2 x\right)}{2x\sin\dfrac{x}{2}\cos\dfrac{x}{2}}=\lim\limits_{x\to 0}\left(\dfrac{1+\cos x+\cos^2 x}{\cos\dfrac{x}{2}}\cdot\dfrac{1}{2}\cdot\dfrac{\sin\dfrac{x}{2}}{\dfrac{x}{2}}\right)=\dfrac{3}{2}.$

18) Ta có $\lim\limits_{x\to a}\dfrac{\sin^2 x-\sin^2 a}{x^2-a^2}=\lim\limits_{x\to a}\dfrac{\dfrac{1-\cos 2x}{2}-\dfrac{1-\cos 2a}{2}}{(x-a)(x+a)}$

$=\lim\limits_{x\to a}\dfrac{\cos 2a-\cos 2x}{2(x-a)(x+a)}=\lim\limits_{x\to a}\dfrac{-2\sin(a+x)\sin(a-x)}{2(x-a)(x+a)}$

$=\lim\limits_{x\to a}\left(\dfrac{\sin(a+x)}{x+a}\cdot\dfrac{\sin(a-x)}{a-x}\right)=\dfrac{\sin 2a}{2a}.$

19) Ta có $\lim\limits_{x\to 0}\dfrac{\cos\alpha x-\cos\beta x}{x^2}=\lim\limits_{x\to 0}\dfrac{-2\sin\dfrac{(\alpha+\beta)x}{2}\sin\dfrac{(\alpha-\beta)x}{2}}{x^2}$

$=\lim\limits_{x\to 0}\left(-2\cdot\dfrac{\alpha+\beta}{2}\cdot\dfrac{\sin\dfrac{(\alpha+\beta)x}{2}}{\dfrac{(\alpha+\beta)x}{2}}\cdot\dfrac{\alpha-\beta}{2}\cdot\dfrac{\sin\dfrac{(\alpha-\beta)x}{2}}{\dfrac{(\alpha-\beta)x}{2}}\right)=\dfrac{\beta^2-\alpha^2}{2}.$

20) Ta có $\lim\limits_{x\to -2}\dfrac{x^3+8}{\tan(x+2)}=\lim\limits_{x\to -2}\dfrac{(x+2)\left(x^2-2x+4\right)}{\tan(x+2)}=\lim\limits_{x\to -2}\left(\left(x^2-2x+4\right)\cdot\dfrac{(x+2)}{\tan(x+2)}\right)=12.$

21) Ta có $\lim\limits_{x\to 0}\dfrac{1-\cos x\cos 2x\cos 3x}{1-\cos x}=\lim\limits_{x\to 0}\dfrac{1-\cos x+\cos x(1-\cos 2x)+\cos x\cos 2x(1-\cos 3x)}{1-\cos x}$

$$=\lim_{x\to 0}\left(1+\cos x\frac{2\sin^2 x}{2\sin^2\frac{x}{2}}+\cos x\cos 2x\frac{2\sin^2\frac{3x}{2}}{2\sin^2\frac{x}{2}}\right)$$

$$=\lim_{x\to 0}\left(1+4\cos x\left(\frac{\sin x}{x}\right)^2\left(\frac{\frac{x}{2}}{\sin\frac{x}{2}}\right)^2+9\cos x\cos 2x\left(\frac{\sin\frac{3x}{2}}{\sin\frac{3x}{2}}\right)^2\left(\frac{\frac{x}{2}}{\sin\frac{x}{2}}\right)^2\right)=1+4+9=14.$$

22) Ta có $\lim_{x\to 0}\frac{\sin(a+2x)-2\sin(a+x)+\sin a}{x^2}=\lim_{x\to 0}\frac{2\sin(a+x)\cos x-2\sin(a+x)}{x^2}$

$$=\lim_{x\to 0}\frac{2\sin(a+x)(\cos x-1)}{x^2}=\lim_{x\to 0}\frac{-4\sin(a+x)\sin^2\frac{x}{2}}{x^2}$$

$$=\lim_{x\to 0}\left(-4\sin(a+x)\cdot\frac{1}{4}\cdot\left(\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right)^2\right)=-\sin a.$$

23) Ta có $\lim_{x\to 0}\frac{\sin ax+\tan bx}{(a+b)x}=\lim_{x\to 0}\frac{ax\cdot\frac{\sin ax}{ax}+bx\cdot\frac{\tan bx}{bx}}{(a+b)x}=\lim_{x\to 0}\frac{a\cdot\frac{\sin ax}{ax}\cdot b\frac{\tan bx}{bx}}{a+b}=\frac{a+b}{a+b}=1.$

24) Ta có $\lim_{x\to 0}\frac{\cos 3x-\cos 5x\cos 7x}{x^2}=\lim_{x\to 0}\frac{\cos 3x-\cos 5x+\cos 5x-\cos 5x\cos 7x}{x^2}$

$$=\lim_{x\to 0}\frac{2\sin 4x\sin x+\cos 5x(1-\cos 7x)}{x^2}=\lim_{x\to 0}\frac{2\sin 4x\sin x-2\cos 5x\sin^2\frac{7x}{2}}{x^2}$$

$$=\lim_{x\to 0}\left(8\cdot\frac{\sin 4x}{4x}\cdot\frac{\sin x}{x}-2\cdot\frac{49}{4}\cdot\cos 5x\cdot\left(\frac{\sin\frac{7x}{2}}{\frac{7x}{2}}\right)^2\right)=8-\frac{49}{2}=-\frac{33}{2}.$$

25) Ta có $\lim_{x\to 0}\frac{\cos ax-\cos bx\cos cx}{x^2}=\lim_{x\to 0}\frac{\cos ax-\cos bx+\cos bx-\cos bx\cos cx}{x^2}$

$$=\lim_{x\to 0}\frac{2\sin\frac{(a+b)x}{2}\sin\frac{(b-a)x}{2}+\cos bx(1-\cos cx)}{x^2}=\lim_{x\to 0}\frac{2\sin\frac{(a+b)x}{2}\sin\frac{(b-a)x}{2}-2\cos bx\sin^2\frac{cx}{2}}{x^2}$$

$$=\lim_{x\to 0}\left(2\cdot\frac{b^2-a^2}{4}\cdot\frac{\sin\frac{(a+b)x}{2}}{\frac{(a+b)x}{2}}\cdot\frac{\sin\frac{(b-a)x}{2}}{\frac{(b-a)x}{2}}-2\cdot\frac{c^2}{4}\cdot\cos bx\cdot\left(\frac{\sin\frac{cx}{2}}{\frac{cx}{2}}\right)^2\right)=\frac{b^2-a^2}{2}-\frac{c^2}{2}=\frac{b^2-a^2-c^2}{2}.$$

26) Ta có $\lim\limits_{x\to 0}\dfrac{\sin(a+x)-\sin(a-x)}{\tan(a+x)-\tan(a-x)}=\lim\limits_{x\to 0}\dfrac{2\cos a\sin x}{\dfrac{\sin 2x}{\cos(a+x)\cos(a-x)}}$

$=\lim\limits_{x\to 0}\dfrac{\cos a\cos(a+x)\cos(a-x)}{\cos x}=\cos^3 a$

27) Ta có $\lim\limits_{x\to 0}\dfrac{\sqrt{2x+1}-\sqrt[3]{x^2+1}}{\cos x}=\lim\limits_{x\to 0}\dfrac{\sqrt{2x+1}-1+1-\sqrt[3]{x^2+1}}{\sin x}$

$=\lim\limits_{x\to 0}\dfrac{\dfrac{2x}{\sqrt{2x+1}+1}+\dfrac{-x^2}{1+\sqrt[3]{x^2+1}+\left(\sqrt[3]{x^2+1}\right)^2}}{\sin x}$

$=\lim\limits_{x\to 0}\dfrac{\dfrac{2x}{\sqrt{2x+1}+1}-\dfrac{x}{1+\sqrt[3]{x^2+1}+\left(\sqrt[3]{x^2+1}\right)^2}}{\dfrac{\sin x}{x}}=1.$

28) Ta có $\lim\limits_{x\to 0}\dfrac{\sin^2 2x-\sin x\cdot\sin 4x}{x^4}=\lim\limits_{x\to 0}\dfrac{\sin 2x\left(\sin 2x-2\sin x\cos 2x\right)}{x^4}$

$=\lim\limits_{x\to 0}\dfrac{2\sin 2x\sin x\left(\cos x-\cos 2x\right)}{x^4}=\lim\limits_{x\to 0}\dfrac{4\sin 2x\sin x\sin\dfrac{3x}{2}\sin\dfrac{x}{2}}{x^4}$

$=\lim\limits_{x\to 0}\left(4\cdot\dfrac{3}{2}\cdot\dfrac{\sin 2x}{2x}\cdot\dfrac{\sin x}{x}\cdot\dfrac{\sin\dfrac{3x}{2}}{\dfrac{3x}{2}}\cdot\dfrac{\sin\dfrac{x}{2}}{\dfrac{x}{2}}\right)=6.$

29) $\lim\limits_{x\to -\frac{\pi}{2}}\dfrac{\cos x}{x+\dfrac{\pi}{2}}=\lim\limits_{x\to -\frac{\pi}{2}}\dfrac{\sin\left(x+\dfrac{\pi}{2}\right)}{x+\dfrac{\pi}{2}}=1.$

30) $\lim\limits_{x\to 0}\dfrac{\sin x-\sin 2x}{x\left(1-2\sin^2\dfrac{x}{2}\right)}=\lim\limits_{x\to 0}\dfrac{\sin x(1-2\cos x)}{x\cos x}=\lim\limits_{x\to 0}\left(\dfrac{\sin x}{x}\cdot\dfrac{1-2\cos x}{\cos x}\right)=-1.$

31) Ta có $\lim\limits_{x\to 0}\dfrac{\sqrt{1+x^2}-\cos x}{x^2}=\lim\limits_{x\to 0}\dfrac{\sqrt{1+x^2}-1+1-\cos x}{x^2}=\lim\limits_{x\to 0}\left(\dfrac{\sqrt{1+x^2}-1}{x^2}+\dfrac{1-\cos x}{x^2}\right)$

$=\lim\limits_{x\to 0}\left(\dfrac{1+x^2-1}{x^2\left(\sqrt{1+x^2}+1\right)}+\dfrac{2\sin^2\dfrac{x}{2}}{x^2}\right)=\lim\limits_{x\to 0}\left(\dfrac{1}{\sqrt{1+x^2}+1}+\dfrac{1}{2}\cdot\left(\dfrac{\sin\dfrac{x}{2}}{\dfrac{x}{2}}\right)^2\right)=\dfrac{1}{2}+\dfrac{1}{2}=1$.

32) $\lim\limits_{x\to 0}\dfrac{\sqrt{1+\tan x}-\sqrt{1+\sin x}}{x^3}=\lim\limits_{x\to 0}\dfrac{1+\tan x-1-\sin x}{x^3\left(\sqrt{1+\tan x}+\sqrt{1+\sin x}\right)}$

$=\lim\limits_{x\to 0}\dfrac{\sin x\left(1-\cos x\right)}{x^3\cos x\left(\sqrt{1+\tan x}+\sqrt{1+\sin x}\right)}=\lim\limits_{x\to 0}\dfrac{2\sin x\sin^2\dfrac{x}{2}}{x^3\cos x\left(\sqrt{1+\tan x}+\sqrt{1+\sin x}\right)}$

$=\lim\limits_{x\to 0}\left(\dfrac{2}{\cos x\left(\sqrt{1+\tan x}+\sqrt{1+\sin x}\right)}\cdot\dfrac{\sin x}{x}\cdot\dfrac{1}{4}\cdot\left(\dfrac{\sin\dfrac{x}{2}}{\dfrac{x}{2}}\right)^2\right)=\dfrac{1}{4}.$

33) $\lim\limits_{x\to 0}\dfrac{1-\cos 5x\cos 7x}{\sin^2 11x}=\lim\limits_{x\to 0}\dfrac{1-\cos 5x+\cos 5x\left(1-\cos 7x\right)}{\sin^2 11x}=\lim\limits_{x\to 0}\left(\dfrac{2\sin^2\dfrac{5x}{2}}{\sin^2 11x}+\dfrac{2\cos 5x\sin^2\dfrac{7x}{2}}{\sin^2 11x}\right)$

$=\lim\limits_{x\to 0}\left(\dfrac{25}{242}\left(\dfrac{\sin\dfrac{5x}{2}}{\dfrac{5x}{2}}\right)^2\left(\dfrac{11x}{\sin 11x}\right)^2+\dfrac{49}{242}\cos 5x\left(\dfrac{\sin\dfrac{7x}{2}}{\dfrac{7x}{2}}\right)^2\cdot\left(\dfrac{11x}{\sin 11x}\right)^2\right)=\dfrac{25}{242}+\dfrac{49}{242}=\dfrac{37}{121}.$

34) $\lim\limits_{x\to 1}\dfrac{\sqrt{x+3}-2}{\tan\left(x-1\right)}=\lim\limits_{x\to 1}\dfrac{x+3-4}{\tan\left(x-1\right)\left(\sqrt{x+3}+2\right)}=\lim\limits_{x\to 1}\left(\dfrac{1}{\sqrt{x+3}+2}\cdot\dfrac{x-1}{\tan\left(x-1\right)}\right)=\dfrac{1}{4}.$

35) $\lim\limits_{x\to\pi}\dfrac{1+\cos x}{\left(x-\pi\right)^2}=\lim\limits_{x\to\pi}\dfrac{2\cos^2\dfrac{x}{2}}{\left(x-\pi\right)^2}=\lim\limits_{x\to\pi}\dfrac{2\sin^2\left(\dfrac{\pi-x}{2}\right)}{\left(x-\pi\right)^2}=\lim\limits_{x\to\pi}\left(\dfrac{1}{2}\cdot\left(\dfrac{\sin\left(\dfrac{\pi-x}{2}\right)}{\dfrac{\pi-x}{2}}\right)^2\right)=\dfrac{1}{2}\ .$

36) $\lim\limits_{x\to 1}\dfrac{\sin\left(x-1\right)}{x^2-4x+3}=\lim\limits_{x\to 1}\dfrac{\sin\left(x-1\right)}{\left(x-1\right)\left(x-3\right)}=\lim\limits_{x\to 1}\left(\dfrac{\sin\left(x-1\right)}{x-1}.\dfrac{1}{x-3}\right)=-\dfrac{1}{2}.$

37) $\lim\limits_{x\to 0}\dfrac{\sqrt{x^2+1}-\cos 2x}{x^2}=\lim\limits_{x\to 0}\dfrac{\sqrt{x^2+1}-1+1-\cos 2x}{x^2}$

$=\lim\limits_{x\to 0}\left(\dfrac{\sqrt{x^2+1}-1}{x^2}+\dfrac{1-\cos 2x}{x^2}\right)=\lim\limits_{x\to 0}\left(\dfrac{x^2+1-1}{x^2\left(\sqrt{x^2+1}+1\right)}+\dfrac{2\sin^2 x}{x^2}\right)$

$=\lim\limits_{x\to 0}\left(\dfrac{1}{\sqrt{x^2+1}+1}+2\cdot\left(\dfrac{\sin x}{x}\right)^2\right)=\dfrac{1}{2}+2=\dfrac{5}{2}\ .$

38) $\lim\limits_{x\to 0}\dfrac{1-\cos x\sqrt{\cos 2x}}{x^2}=\lim\limits_{x\to 0}\dfrac{1-\cos x+\cos x\left(1-\sqrt{\cos 2x}\right)}{x^2}$

$$=\lim_{x\to 0}\left(\frac{1-\cos x}{x^2}+\frac{\cos x\left(1-\sqrt{\cos 2x}\right)}{x^2}\right)=\lim_{x\to 0}\left(\frac{2\sin^2\frac{x}{2}}{x^2}+\frac{\cos x\left(1-\cos 2x\right)}{x^2\left(1+\sqrt{\cos 2x}\right)}\right)$$

$$=\lim_{x\to 0}\left(\frac{1}{2}\cdot\left(\frac{\sin\frac{x}{2}}{x^2\left(1+\sqrt{\cos 2x}\right)}\right)=\lim_{x\to 0}\left(\frac{1}{2}\cdot\left(\frac{\sin\frac{x}{2}}{1+\sqrt{\cos 2x}}\cdot\left(\frac{\sin x}{x}\right)^2\right)^2\right)=\frac{1}{2}+1=\frac{3}{2}\right.$$

Bài 9. Tính các giới hạn sau:

1) $\lim\limits_{x\to 0}\frac{\cos 3x-\cos x}{\cos 5x-\cos x}$ ĐS: $\frac{1}{3}$

2) $\lim\limits_{x\to\frac{\pi}{6}}\frac{1-2\sin x}{4\cos^2 x-3}$ ĐS: $\frac{1}{2}$

3) $\lim\limits_{x\to\frac{\pi}{2}}\frac{1+\sin 2x+\cos 2x}{\cos x}$ ĐS:

24) $\lim\limits_{x\to 0}\frac{\sin 7x-\sin 5x}{\sin x}$ ĐS: 2

5) $\lim\limits_{x\to\frac{\pi}{4}}\frac{\sqrt{2}-2\cos x}{\sin\left(x-\frac{\pi}{4}\right)}$ ĐS: $\sqrt{2}$

6) $\lim\limits_{x\to 0}\frac{1-\sqrt[3]{\cos x}}{\sin x}$ ĐS: $\frac{1}{6}$

7) $\lim\limits_{x\to\frac{\pi}{3}}\frac{\sin x-\sqrt[3]{\cos x}}{\sin 3x}$ ĐS: $-\frac{2}{3}$

8) $\lim\limits_{x\to\frac{\pi}{4}}\left[\tan 2x.\tan\left(\frac{\pi}{4}-x\right)\right]$ ĐS: 1

9) $\lim\limits_{x\to\frac{\pi}{3}}\frac{\cos 3x+2\cos 2x+2}{\sin 3x}$ ĐS: $\frac{2\sqrt{3}}{3}$

10) $\lim\limits_{x\to\frac{\pi}{3}}\frac{\sqrt[3]{\tan x}-1}{2\sin^2 x-1}$ ĐS: $-\frac{1}{12}$

Lời giải

1) $\lim\limits_{x\to 0}\frac{\cos 3x-\cos x}{\cos 5x-\cos x}=\lim\limits_{x\to 0}\frac{-2\sin 2x\sin x}{-2\sin 3x\sin 2x}=\lim\limits_{x\to 0}\frac{\sin x}{\sin 3x}=\lim\limits_{x\to 0}\frac{\frac{\sin x}{x}}{3.\frac{\sin 3x}{3x}}=\frac{1}{3}$.

2) $\lim\limits_{x\to\frac{\pi}{6}}\frac{1-2\sin x}{4\cos^2 x-3}=\lim\limits_{x\to\frac{\pi}{6}}\frac{1-2\sin x}{1-4\sin^2 x}=\lim\limits_{x\to\frac{\pi}{6}}\frac{1}{1+2\sin x}=\frac{1}{2}$.

3) $\lim\limits_{x\to\frac{\pi}{2}}\frac{1+\sin 2x+\cos 2x}{\cos x}=\lim\limits_{x\to\frac{\pi}{2}}\frac{2\cos^2 x+\sin 2x}{\cos x}=\lim\limits_{x\to\frac{\pi}{2}}\left(2\cos x+2\sin x\right)=2$.

4) $\lim\limits_{x\to 0}\frac{\sin 7x-\sin 5x}{\sin x}=\lim\limits_{x\to 0}\frac{2\cos 6x\sin x}{\sin x}=\lim\limits_{x\to 0}2\cos 6x=2$.

5) $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\sqrt{2}-2\cos x}{\sin\left(x-\dfrac{\pi}{4}\right)}=\lim\limits_{x\to\frac{\pi}{4}}\dfrac{2\left(\dfrac{\sqrt{2}}{2}-\cos x\right)}{\sin\left(x-\dfrac{\pi}{4}\right)}=\lim\limits_{x\to\frac{\pi}{4}}\dfrac{2\left(\cos\dfrac{\pi}{4}-\cos x\right)}{\sin\left(x-\dfrac{\pi}{4}\right)}$

$=\lim\limits_{x\to\frac{\pi}{4}}\dfrac{-4\sin\left(\dfrac{\pi}{8}+\dfrac{x}{2}\right)\sin\left(\dfrac{\pi}{8}-\dfrac{x}{2}\right)}{2\sin\left(\dfrac{x}{2}-\dfrac{\pi}{8}\right)\cos\left(\dfrac{x}{2}-\dfrac{\pi}{8}\right)}=\lim\limits_{x\to\frac{\pi}{4}}\dfrac{2\sin\left(\dfrac{\pi}{8}+\dfrac{x}{2}\right)}{\cos\left(\dfrac{x}{2}-\dfrac{\pi}{8}\right)}=\sqrt{2}$

6) $\lim\limits_{x\to0}\dfrac{1-\sqrt[3]{\cos x}}{\tan^2 x}=\lim\limits_{x\to0}\dfrac{(1-\cos x)\cdot\cos^2 x}{\sin^2 x\cdot\left(1+\sqrt[3]{\cos x}+\sqrt[3]{\cos^2 x}\right)}$

$=\lim\limits_{x\to0}\dfrac{\cos^2 x}{(1+\cos x)\cdot\left(1+\sqrt[3]{\cos x}+\sqrt[3]{\cos^2 x}\right)}=\dfrac{1}{6}$

7) $\lim\limits_{x\to\frac{\pi}{3}}\dfrac{\sin x-\sqrt{3}\cos x}{\sin 3x}=\lim\limits_{x\to\frac{\pi}{3}}\dfrac{2\sin\left(x-\dfrac{\pi}{3}\right)}{\sin(3x-\pi+\pi)}=\lim\limits_{x\to\frac{\pi}{3}-\frac{\pi}{3}-\sin\left[3\left(x-\frac{\pi}{3}\right)\right]}=\lim\limits_{x\to\frac{\pi}{3}}\dfrac{\dfrac{2\sin\left[3\left(x-\dfrac{\pi}{3}\right)\right]}{\sin\left[3\left(x-\dfrac{\pi}{3}\right)\right]}}{-3\cdot\dfrac{\sin\left[3\left(x-\dfrac{\pi}{3}\right)\right]}{3\left(x-\dfrac{\pi}{3}\right)}}=-\dfrac{2}{3}.$

8) $\lim\limits_{x\to\frac{\pi}{4}}\left[\tan 2x\cdot\tan\left(\dfrac{\pi}{4}-x\right)\right]=\lim\limits_{x\to\frac{\pi}{4}}\left[\dfrac{2\tan x}{1-\tan^2 x}\cdot\dfrac{1-\tan x}{1+\tan x}\right]=\lim\limits_{x\to\frac{\pi}{4}}\dfrac{2\tan x}{(1+\tan x)^2}=1.$

9) $\lim\limits_{x\to\frac{\pi}{3}}\dfrac{\cos 3x+2\cos 2x+2}{\sin 3x}=\lim\limits_{x\to\frac{\pi}{3}}\dfrac{4\cos^3 x-3\cos x+4\cos^2 x}{3\sin x-4\sin^3 x}$

$=\lim\limits_{x\to\frac{\pi}{3}}\dfrac{(2\cos x-1)(2\cos x+3)\cos x}{\left(2\sin x+\sqrt{3}\right)\left(2\sin x-\sqrt{3}\right)\sin x}=\lim\limits_{x\to\frac{\pi}{3}}\dfrac{\left(\cos x-\cos\dfrac{\pi}{3}\right)(2\cos x+3)\cos x}{3}$

$=\lim\limits_{x\to\frac{\pi}{3}}\dfrac{-\sin\left(\dfrac{x}{2}+\dfrac{\pi}{6}\right)(2\cos x+3)\cos x}{\cos\left(\dfrac{x}{2}+\dfrac{\pi}{6}\right)\left(2\sin x+\sqrt{3}\right)\sin x}=\dfrac{2\sqrt{3}}{3}$

10) $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\sqrt[3]{\tan x}-1}{2\sin^2 x-1}=\lim\limits_{x\to\frac{\pi}{4}}\dfrac{(\tan x-1)\cos^2 x}{(1-\tan^2 x).\left[\left(\sqrt[3]{\tan x}\right)^2+\sqrt[3]{\tan x}+1\right]}$

$=\lim\limits_{x\to\frac{\pi}{4}}\dfrac{-\cos^2 x}{(1+\tan x)\left[\left(\sqrt[3]{\tan x}\right)^2+\sqrt[3]{\tan x}+1\right]}=-\dfrac{1}{12}.$

Bài 10. Tính các giới hạn sau:

1) $\lim_{x\to 0}\frac{\cos 4x-1}{\sin 4x}$ **ĐS:** 0

2) $\lim_{x\to 0}\frac{1+\sin 2x-\cos 2x}{1-\sin 2x-\cos 2x}$ **ĐS:** -1

3) $\lim_{x\to 0}\frac{\sin 2x}{1-\sin 2x-\cos 2x}$ **ĐS:** -1

4) $\lim_{x\to 0}\frac{1-\cos x}{\sin x}$ **ĐS:** 0

5) $\lim_{x\to 0}\frac{\sin 5x-\sin 3x}{\sin x}$ **ĐS:** 2

6) $\lim_{x\to 0}\left(\frac{1}{\sin x}-\frac{1}{\tan x}\right)$ **ĐS:** 0

7) $\lim_{x\to \frac{\pi}{4}}\frac{\sqrt{2}\sin x-1}{2\cos^2 x-1}$ **ĐS:** $-\frac{\sqrt{2}}{4}$

8) $\lim_{x\to \frac{\pi}{6}}\frac{\sin\left(\frac{\pi}{6}-x\right)}{1-2\sin x}$ **ĐS:** $\frac{2\sqrt{3}}{3}$

9) $\lim_{x\to \frac{\pi}{4}}\frac{\sin\left(\frac{\pi}{4}-x\right)}{1-\sqrt{2}\sin x}$ **ĐS:** $\sqrt{2}$

10) $\lim_{x\to 0}\left(\frac{2}{\sin 2x}-\cot x\right)$ **ĐS: 0**