

TP1: TÍCH PHÂN HÀM SỐ HỮU TỈ

Dạng 1: Tách phân thức

Câu 1. $I = \int_1^2 \frac{x^2}{x^2 - 7x + 12} dx$

• $I = \int_1^2 \left(1 + \frac{16}{x-4} - \frac{9}{x-3} \right) dx = (x + 16\ln|x-4| - 9\ln|x-3|) \Big|_1^2 = 1 + 25\ln 2 - 16\ln 3.$

Câu 2. $I = \int_1^2 \frac{dx}{x^5 + x^3}$

• Ta có: $\frac{1}{x^3(x^2+1)} = -\frac{1}{x} + \frac{1}{x^3} + \frac{x}{x^2+1}$

$\Rightarrow I = \left[-\ln|x| - \frac{1}{2x^2} + \frac{1}{2}\ln(x^2+1) \right] \Big|_1^2 = -\frac{3}{2}\ln 2 + \frac{1}{2}\ln 5 + \frac{3}{8}$

Câu 3. $I = \int_4^5 \frac{3x^2+1}{x^3-2x^2-5x+6} dx$

• $I = -\frac{2}{3}\ln\frac{4}{3} + \frac{13}{15}\ln\frac{7}{6} + \frac{14}{5}\ln 2$

Câu 4. $I = \int_0^1 \frac{xdx}{(x+1)^3}$

• Ta có: $\frac{x}{(x+1)^3} = \frac{x+1-1}{(x+1)^3} = (x+1)^{-2} - (x+1)^{-3} \Rightarrow I = \int_0^1 [(x+1)^{-2} - (x+1)^{-3}] dx = \frac{1}{8}$

Dạng 2: Đổi biến số

Câu 5. $I = \int \frac{(x-1)^2}{(2x+1)^4} dx$ • Ta có: $f(x) = \frac{1}{3} \cdot \left(\frac{x-1}{2x+1} \right)^2 \cdot \left(\frac{x-1}{2x+1} \right)' \Rightarrow I = \frac{1}{9} \left(\frac{x-1}{2x+1} \right)^3 + C$

Câu 6. $I = \int_0^1 \frac{(7x-1)^{99}}{(2x+1)^{101}} dx$

• $I = \int_0^1 \left(\frac{7x-1}{2x+1} \right)^{99} \frac{dx}{(2x+1)^2} = \frac{1}{9} \int_0^1 \left(\frac{7x-1}{2x+1} \right)^{99} d\left(\frac{7x-1}{2x+1} \right)$
 $= \frac{1}{9} \cdot \frac{1}{100} \left(\frac{7x-1}{2x+1} \right)^{100} \Big|_0^1 = \frac{1}{900} [2^{100} - 1]$

Câu 7. $I = \int_0^1 \frac{5x}{(x^2+4)^2} dx$

• Đặt $t = x^2 + 4 \Rightarrow I = \frac{1}{8}$

Câu 8. $I = \int_0^1 \frac{x^7}{(1+x^2)^5} dx$

• Đặt $t = 1 + x^2 \Rightarrow dt = 2xdx \Rightarrow I = \frac{1}{2} \int_1^2 \frac{(t-1)^3}{t^5} dt = \frac{1}{4} \cdot \frac{1}{2^5}$

Câu 9. $I = \int_0^1 x^5(1-x^3)^6 dx$

• Đặt $t = 1 - x^3 \Rightarrow dt = -3x^2 dx \Rightarrow dx = \frac{-dt}{3x^2} \Rightarrow I = \frac{1}{3} \int_0^1 t^6(1-t) dt = \frac{1}{3} \left(\frac{t^7}{7} - \frac{t^8}{8} \right) = \frac{1}{168}$

Câu 10. $I = \int_1^{\sqrt[4]{3}} \frac{1}{x(x^4+1)} dx$

• Đặt $t = x^4 \Rightarrow I = \frac{1}{2} \int_1^{\sqrt[4]{3}} \left(\frac{1}{t} - \frac{t}{t^2+1} \right) dt = \frac{1}{4} \ln \frac{3}{2}$

Câu 11. $I = \int_1^2 \frac{dx}{x \cdot (x^{10}+1)^2}$

• $I = \int_1^2 \frac{x^4 \cdot dx}{x^5 \cdot (x^{10}+1)^2}$. Đặt $t = x^5 \Rightarrow I = \frac{1}{5} \int_1^{32} \frac{dt}{t(t^2+1)^2}$

Câu 12. $I = \int_1^2 \frac{1-x^7}{x(1+x^7)} dx$

• $I = \int_1^2 \frac{(1-x^7) \cdot x^6}{x^7 \cdot (1+x^7)} dx$. Đặt $t = x^7 \Rightarrow I = \frac{1}{7} \int_1^{128} \frac{1-t}{t(1+t)} dt$

Câu 13. $I = \int_1^{\sqrt{3}} \frac{dx}{x^6(1+x^2)}$

• Đặt $x = \frac{1}{t} \Rightarrow I = - \int_1^{\frac{\sqrt{3}}{3}} \frac{t^6}{t^2+1} dt = \int_{\frac{\sqrt{3}}{3}}^1 \left(t^4 - t^2 + 1 - \frac{1}{t^2+1} \right) dt = \frac{117-41\sqrt{3}}{135} + \frac{\pi}{12}$

Câu 14. $I = \int_1^2 \frac{x^{2001}}{(1+x^2)^{1002}} dx$

• $I = \int_1^2 \frac{x^{2004}}{x^3(1+x^2)^{1002}} dx = \int_1^2 \frac{1}{x^3 \left(\frac{1}{x^2} + 1 \right)^{1002}} dx$. Đặt $t = \frac{1}{x^2} + 1 \Rightarrow dt = -\frac{2}{x^3} dx$.

Cách 2: Ta có: $I = \frac{1}{2} \int_0^1 \frac{x^{2000} \cdot 2x dx}{(1+x^2)^{2000}(1+x^2)^2}$. Đặt $t = 1+x^2 \Rightarrow dt = 2x dx$

$\Rightarrow I = \frac{1}{2} \int_1^2 \frac{(t-1)^{1000}}{t^{1000}t^2} dt = \frac{1}{2} \int_1^2 \left(1 - \frac{1}{t} \right)^{1000} d \left(1 - \frac{1}{t} \right) = \frac{1}{2002 \cdot 2^{1001}}$

Câu 15. $I = \int_1^2 \frac{1+x^2}{1+x^4} dx$

• Ta có: $\frac{1+x^2}{1+x^4} = \frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}}$. Đặt $t = x - \frac{1}{x} \Rightarrow dt = \left(1 + \frac{1}{x^2} \right) dx$

$\Rightarrow I = \int_1^2 \frac{\frac{3}{2} dt}{t^2-2} = \frac{1}{2\sqrt{2}} \int_1^{\frac{3}{2}} \left(\frac{1}{t-\sqrt{2}} - \frac{1}{t+\sqrt{2}} \right) dt = \frac{1}{2\sqrt{2}} \cdot \ln \left| \frac{t-\sqrt{2}}{t+\sqrt{2}} \right| \Big|_1^{\frac{3}{2}} = \frac{1}{2\sqrt{2}} \ln \left(\frac{\sqrt{2}-1}{\sqrt{2}+1} \right)$

Câu 16. $I = \int_1^2 \frac{1-x^2}{1+x^4} dx$

• Ta có: $\frac{1-x^2}{1+x^4} = \frac{\frac{1}{x^2}-1}{x^2+\frac{1}{x^2}}$. Đặt $t = x + \frac{1}{x} \Rightarrow dt = \left(1 - \frac{1}{x^2}\right) dx \Rightarrow I = -\int_2^{\frac{5}{2}} \frac{dt}{t^2+2}$.

Đặt $t = \sqrt{2} \tan u \Rightarrow dt = \sqrt{2} \frac{du}{\cos^2 u}$; $\tan u = 2 \Rightarrow u_1 = \arctan 2$; $\tan u = \frac{5}{2} \Rightarrow u_2 = \arctan \frac{5}{2}$

$\Rightarrow I = \frac{\sqrt{2}}{2} \int_{u_1}^{u_2} du = \frac{\sqrt{2}}{2} (u_2 - u_1) = \frac{\sqrt{2}}{2} \left(\arctan \frac{5}{2} - \arctan 2 \right)$

Câu 17. $I = \int_1^2 \frac{1-x^2}{x+x^3} dx$ • Ta có: $I = \int_1^2 \frac{\frac{1}{x^2}-1}{\frac{1}{x}+x} dx$. Đặt $t = x + \frac{1}{x} \Rightarrow I = \ln \frac{4}{5}$

Câu 18. $I = \int_0^1 \frac{x^4+1}{x^6+1} dx$

• Ta có: $\frac{x^4+1}{x^6+1} = \frac{(x^4-x^2+1)+x^2}{x^6+1} = \frac{x^4-x^2+1}{(x^2+1)(x^4-x^2+1)} + \frac{x^2}{x^6+1} = \frac{1}{x^2+1} + \frac{x^2}{x^6+1}$

$\Rightarrow I = \int_0^1 \frac{1}{x^2+1} dx + \frac{1}{3} \int_0^1 \frac{d(x^3)}{(x^3)^2+1} dx = \frac{\pi}{4} + \frac{1}{3} \cdot \frac{\pi}{4} = \frac{\pi}{3}$

Câu 19. $I = \int_0^{\frac{\sqrt{3}}{3}} \frac{x^2}{x^4-1} dx$

• $I = \int_0^{\frac{\sqrt{3}}{3}} \frac{x^2}{(x^2-1)(x^2+1)} dx = \frac{1}{2} \int_0^{\frac{\sqrt{3}}{3}} \left(\frac{1}{x^2-1} + \frac{1}{x^2+1} \right) dx = \frac{1}{4} \ln(2-\sqrt{3}) + \frac{\pi}{12}$

Câu 20. $I = \int_0^1 \frac{xdx}{x^4+x^2+1}$. • Đặt $t = x^2 \Rightarrow I = \frac{1}{2} \int_0^1 \frac{dt}{t^2+t+1} = \frac{1}{2} \int_0^1 \frac{dt}{\left(t+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \frac{\pi}{6\sqrt{3}}$

Câu 21. $I = \int_1^{\frac{1+\sqrt{5}}{2}} \frac{x^2+1}{x^4-x^2+1} dx$

• Ta có: $\frac{x^2+1}{x^4-x^2+1} = \frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}-1}$. Đặt $t = x - \frac{1}{x} \Rightarrow dt = \left(1 + \frac{1}{x^2}\right) dx$

$\Rightarrow I = \int_0^1 \frac{dt}{t^2+1}$. Đặt $t = \tan u \Rightarrow dt = \frac{du}{\cos^2 u} \Rightarrow I = \int_0^{\frac{\pi}{4}} du = \frac{\pi}{4}$