TP3: TÍCH PHÂN HÀM SỐ LƯỢNG GIÁC

Dạng 1: Biến đổi lượng giác

Câu 1. $I=\int \frac{8\cos^2 x-\sin 2x-3}{\sin x-\cos x}dx$

• $I=\int \frac{(\sin x-\cos x)^2+4\cos 2x}{\sin x-\cos x}dx=\int\left[\left(\sin x-\cos x-4(\sin x+\cos x\right)\right]dx$
$=3\cos x-5\sin x+C.$

Câu 2. $I=\int \frac{\cot x-\tan x-2\tan 2x}{\sin 4x}dx$

• *Ta có:* $I=\int \frac{2\cot 2x-2\tan 2x}{\sin 4x}dx=\int \frac{2\cot 4x}{\sin 4x}dx=2\int \frac{\cos 4x}{\sin^2 4x}dx=-\frac{1}{2\sin 4x}+C$

Câu 3. $I=\int \frac{\cos^2\left(x+\frac{\pi}{8}\right)}{\sin 2x+\cos 2x+\sqrt{2}}dx$

• *Ta có:* $I=\frac{1}{2\sqrt{2}}\int \frac{1+\cos\left(2x+\frac{\pi}{4}\right)}{1+\sin\left(2x+\frac{\pi}{4}\right)}dx$

$$=\frac{1}{2\sqrt{2}}\left(\int \frac{\cos\left(2x+\frac{\pi}{4}\right)}{1+\sin\left(2x+\frac{\pi}{4}\right)}dx+\int \frac{dx}{\left[\sin\left(x+\frac{\pi}{8}\right)+\cos\left(x+\frac{\pi}{8}\right)\right]^2}\right)$$

$$=\frac{1}{2\sqrt{2}}\left(\int \frac{\cos\left(2x+\frac{\pi}{4}\right)}{1+\sin\left(2x+\frac{\pi}{4}\right)}dx+\frac{1}{2}\int \frac{dx}{\sin^2\left(x+\frac{3\pi}{8}\right)}\right)$$

$$=\frac{1}{4\sqrt{2}}\left(\ln\left|1+\sin\left(2x+\frac{\pi}{4}\right)\right|-\cot\left(x+\frac{3\pi}{8}\right)\right)+C$$

Câu 4. $I=\int_{\frac{\pi}{3}}^{\pi}\frac{dx}{2+\sqrt{3}\sin x-\cos x}$

• $I=\frac{1}{2}\int_{\frac{\pi}{3}}^{\pi}\frac{dx}{1-\cos\left(x+\frac{\pi}{3}\right)}=I=\frac{1}{4}\int_{\frac{\pi}{3}}^{\pi}\frac{dx}{2\sin^2\left(\frac{x}{2}+\frac{\pi}{6}\right)}=\frac{1}{4\sqrt{3}}.$

Câu 5. $I=\int_0^{\frac{\pi}{6}}\frac{1}{2\sin x-\sqrt{3}}dx$

• *Ta có:* $I=\frac{1}{2}\int_0^{\frac{\pi}{6}}\frac{1}{\sin x-\sin\frac{\pi}{3}}dx=\int_0^{\frac{\pi}{6}}\frac{\frac{1}{2}}{\sin x-\sin\frac{\pi}{3}}dx$

$$= \int_0^{\frac{\pi}{6}} \frac{\cos\frac{\pi}{3}}{\sin x - \sin\frac{\pi}{3}} dx = \int_0^{\frac{\pi}{6}} \frac{\cos\left(\left(\frac{x}{2}+\frac{\pi}{6}\right)-\left(\frac{x}{2}-\frac{\pi}{6}\right)\right)}{2\cos\left(\frac{x}{2}+\frac{\pi}{6}\right).\sin\left(\frac{x}{2}-\frac{\pi}{6}\right)} dx$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{6}} \frac{\cos\left(\frac{x}{2}-\frac{\pi}{6}\right)}{\sin\left(\frac{x}{2}-\frac{\pi}{6}\right)} dx + \frac{1}{2}\int_0^{\frac{\pi}{6}} \frac{\sin\left(\frac{x}{2}+\frac{\pi}{6}\right)}{\cos\left(\frac{x}{2}+\frac{\pi}{6}\right)} dx = \ln\left|\sin\left(\frac{x}{2}-\frac{\pi}{6}\right)\right| \Big|_0^{\frac{\pi}{6}} - \ln\left|\cos\left(\frac{x}{2}+\frac{\pi}{6}\right)\right| \Big|_0^{\frac{\pi}{6}} = ...$$

Câu 6. $I = \int_0^{\frac{\pi}{2}} (\sin^4 x + \cos^4 x)(\sin^6 x + \cos^6 x) dx$.

• *Ta có:* $(\sin^4 x + \cos^4 x)(\sin^6 x + \cos^6 x) = \frac{33}{64} + \frac{7}{16}\cos 4x + \frac{3}{64}\cos 8x \Rightarrow I = \frac{33}{128}\pi$.

Câu 7. $I = \int_0^{\frac{\pi}{2}} \cos 2x(\sin^4 x + \cos^4 x) dx$

• $I = \int_0^{\frac{\pi}{2}} \cos 2x\left(1 - \frac{1}{2}\sin^2 2x\right) dx = \frac{1}{2}\int_0^{\frac{\pi}{2}} \left(1 - \frac{1}{2}\sin^2 2x\right) d(\sin 2x) = 0$

Câu 8. $I = \int_0^{\frac{\pi}{2}} (\cos^3 x - 1)\cos^2 x.dx$

• $A = \int_0^{\frac{\pi}{2}} \cos^5 x dx = \int_0^{\frac{\pi}{2}} \left(1 - \sin^2 x\right)^2 d(\sin x) = \frac{8}{15}$

$B = \int_0^{\frac{\pi}{2}} \cos^2 x.dx = \frac{1}{2}\int_0^{\frac{\pi}{2}} (1 + \cos 2x).dx = \frac{\pi}{4}$

Vậy $I = \frac{8}{15} - \frac{\pi}{4}$.

Câu 9. $\mathrm{I} = \int_0^{\frac{\pi}{2}} \cos^2 x \cos 2x dx$

• $I = \int_0^{\frac{\pi}{2}} \cos^2 x \cos 2x dx = \frac{1}{2}\int_0^{\frac{\pi}{2}} (1 + \cos 2x)\cos 2x dx = \frac{1}{4}\int_0^{\frac{\pi}{2}} (1 + 2\cos 2x + \cos 4x) dx$

$$= \frac{1}{4}\left(x + \sin 2x + \frac{1}{4}\sin 4x\right)\Big|_0^{\frac{\pi}{2}} = \frac{\pi}{8}$$

Câu 10. $I = \int_0^{\frac{\pi}{2}} \frac{4\sin^3 x}{1 + \cos x} dx$

- $\dfrac{4\sin^3 x}{1+\cos x} = \dfrac{4\sin^3 x(1-\cos x)}{\sin^2 x} = 4\sin x - 4\sin x\cos x = 4\sin x - 2\sin 2x$

$\Rightarrow I = \int_0^{\frac{\pi}{2}} (4\sin x - 2\sin 2x)dx = 2$

Câu 11. $I = \int\limits_0^{2\pi} \sqrt{1+\sin x}\,dx$

- $I = \int\limits_0^{2\pi} \sqrt{\left(\sin\dfrac{x}{2}+\cos\dfrac{x}{2}\right)^2}dx = \int\limits_0^{2\pi} \left|\sin\dfrac{x}{2}+\cos\dfrac{x}{2}\right|dx = \sqrt{2}\int\limits_0^{2\pi}\left|\sin\left(\dfrac{x}{2}+\dfrac{\pi}{4}\right)\right|dx$

$= \sqrt{2}\left[\int\limits_0^{\frac{3\pi}{2}} \sin\left(\dfrac{x}{2}+\dfrac{\pi}{4}\right)dx - \int\limits_{\frac{3\pi}{2}}^{2\pi} \sin\left(\dfrac{x}{2}+\dfrac{\pi}{4}\right)dx\right] = 4\sqrt{2}$

Câu 12. $I = \int\limits_0^{\frac{\pi}{4}} \dfrac{dx}{\cos^6 x}$ • *Ta có:* $I = \int\limits_0^{\frac{\pi}{4}} (1+2\tan^2 x+\tan^4 x)d(\tan x) = \dfrac{28}{15}$.

Dạng 2: Đổi biến số dạng 1

Câu 13. $I = \int \dfrac{\sin 2x dx}{3+4\sin x-\cos 2x}$

- *Ta có:* $I = \int \dfrac{2\sin x\cos x}{2\sin^2 x+4\sin x+2}dx$. *Đặt* $t = \sin x \Rightarrow I = \ln|\sin x+1| + \dfrac{1}{\sin x+1} + C$

Câu 14. $I = \int \dfrac{dx}{\sin^3 x.\cos^5 x}$

- $I = \int \dfrac{dx}{\sin^3 x.\cos^3 x.\cos^2 x} = 8\int \dfrac{dx}{\sin^3 2x.\cos^2 x}$

Đặt $t = \tan x$. $I = \int\left(t^3+3t+\dfrac{3}{t}+t^{-3}\right)dt = \dfrac{1}{4}\tan^4 x + \dfrac{3}{2}\tan^2 x + 3\ln|\tan x| - \dfrac{1}{2\tan^2 x} + C$

Chú ý: $\sin 2x = \dfrac{2t}{1+t^2}$.

Câu 15. $I = \int \dfrac{dx}{\sin x.\cos^3 x}$

- $I = \int \dfrac{dx}{\sin x.\cos x.\cos^2 x} = 2\int \dfrac{dx}{\sin 2x.\cos^2 x}$. *Đặt* $t = \tan x \Rightarrow dt = \dfrac{dx}{\cos^2 x}$; $\sin 2x = \dfrac{2t}{1+t^2}$

$\Rightarrow I = 2\int \dfrac{dt}{\dfrac{2t}{1+t^2}} = \int \dfrac{t^2+1}{t}dt = \int (t+\dfrac{1}{t})dt = \dfrac{t^2}{2} + \ln|t| + C = \dfrac{\tan^2 x}{2} + \ln|\tan x| + C$

Câu 16. $I = \int \frac{\sqrt[2011]{\sin^{2011} x - \sin^{2009} x}}{\sin^5 x} \cot x dx$

• *Ta có:* $I = \int \frac{\sqrt[2011]{1 - \frac{1}{\sin^2 x}}}{\sin^4 x} \cot x dx = \int \frac{\sqrt[2011]{-\cot^2 x}}{\sin^4 x} \cot x dx$

Đặt $t = \cot x \Rightarrow I = \int t^{\frac{2}{2011}}(1+t^2)tdt = \frac{2011}{4024} t^{\frac{4024}{2011}} + \frac{2011}{8046} t^{\frac{8046}{2011}} + C$

$= \frac{2011}{4024} \cot^{\frac{4024}{2011}} x + \frac{2011}{8046} \cot^{\frac{8046}{2011}} x + C$

Câu 17. $I = \int_0^{\frac{\pi}{2}} \frac{\sin 2x.\cos x}{1+\cos x} dx$

• *Ta có:* $I = 2\int_0^{\frac{\pi}{2}} \frac{\sin x.\cos^2 x}{1+\cos x} dx$. *Đặt* $t = 1+\cos x \Rightarrow I = 2\int_1^2 \frac{(t-1)^2}{t} dt = 2\ln 2 - 1$

Câu 18. $I = \int_0^{\frac{\pi}{3}} \sin^2 x \tan x dx$

• *Ta có:* $I = \int_0^{\frac{\pi}{3}} \sin^2 x.\frac{\sin x}{\cos x} dx = \int_0^{\frac{\pi}{3}} \frac{(1-\cos^2 x)\sin x}{\cos x} dx$. *Đặt* $t = \cos x$

$\Rightarrow I = -\int_1^{\frac{1}{2}} \frac{1-u^2}{u} du = \ln 2 - \frac{3}{8}$

Câu 19. $I = \int_{\frac{\pi}{2}}^{\pi} \sin^2 x(2 - \sqrt{1+\cos 2x})dx$

• *Ta có:* $I = \int_{\frac{\pi}{2}}^{\pi} 2\sin^2 x dx - \int_{\frac{\pi}{2}}^{\pi} \sin^2 x\sqrt{1+\cos 2x} dx = H + K$

+ $H = \int_{\frac{\pi}{2}}^{\pi} 2\sin^2 x dx = \int_{\frac{\pi}{2}}^{\pi} (1-\cos 2x)dx = \pi - \frac{\pi}{2} = \frac{\pi}{2}$

+ $K = \int_{\frac{\pi}{2}}^{\pi} \sin^2 x\sqrt{2\cos^2 x} = -\sqrt{2}\int_{\frac{\pi}{2}}^{\pi} \sin^2 x \cos x dx = -\sqrt{2}\int_{\frac{\pi}{2}}^{\pi} \sin^2 x d(\sin x) = \frac{\sqrt{2}}{3}$

$\Rightarrow I = \frac{\pi}{2} - \frac{\sqrt{2}}{3}$

Câu 20. $I=\int\limits_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{dx}{\sin^2 x.\cos^4 x}$

• $I=4.\int\limits_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{dx}{\sin^2 2x.\cos^2 x}$. *Đặt* $t=\tan x \Rightarrow dt=\frac{dx}{\cos^2 x}$.

$I=\int\limits_{1}^{\sqrt{3}} \frac{(1+t^2)^2 dt}{t^2}=\int\limits_{1}^{\sqrt{3}} \left(\frac{1}{t^2}+2+t^2\right)dt=\left.\left(-\frac{1}{t}+2t+\frac{t^3}{3}\right)\right|_1^{\sqrt{3}}=\frac{8\sqrt{3}-4}{3}$

Câu 21. $I=\int\limits_{0}^{\frac{\pi}{2}} \frac{\sin 2x}{(2+\sin x)^2}dx$

• *Ta có:* $I=\int\limits_{0}^{\frac{\pi}{2}} \frac{\sin 2x}{(2+\sin x)^2}dx=2\int\limits_{0}^{\frac{\pi}{2}} \frac{\sin x\cos x}{(2+\sin x)^2}dx$. *Đặt* $t=2+\sin x$.

$\Rightarrow I=2\int\limits_{2}^{3} \frac{t-2}{t^2}dt=2\int\limits_{2}^{3}\left(\frac{1}{t}-\frac{2}{t^2}\right)dt=2\left.\left(\ln t+\frac{2}{t}\right)\right|_2^3=2\ln\frac{3}{2}-\frac{2}{3}$

Câu 22. $I=\int\limits_{0}^{\frac{\pi}{6}} \frac{\sin x}{\cos 2x}dx$

• $I=\int\limits_{0}^{\frac{\pi}{6}} \frac{\sin x}{\cos 2x}dx=\int\limits_{0}^{\frac{\pi}{6}} \frac{\sin x}{2\cos^2 x-1}dx$. *Đặt* $t=\cos x \Rightarrow dt=-\sin x dx$

Đổi cận: $x=0 \Rightarrow t=1; \quad x=\frac{\pi}{6} \Rightarrow t=\frac{\sqrt{3}}{2}$

Ta được $I=-\int\limits_{1}^{\frac{\sqrt{3}}{2}} \frac{1}{2t^2-1}dt=\frac{1}{2\sqrt{2}}\ln\left.\left|\frac{2t-\sqrt{2}}{2t+\sqrt{2}}\right|\right|_{\frac{\sqrt{3}}{2}}^{1}=\frac{1}{2\sqrt{2}}\ln\left|\frac{3-2\sqrt{2}}{5-2\sqrt{6}}\right|$

Câu 23. $I=\int\limits_{0}^{\frac{\pi}{2}} e^{\sin^2 x}.\sin x.\cos^3 x.\,dx$ • *Đặt* $t=\sin^2 x \Rightarrow I=\frac{1}{2}\int\limits_{0}^{1} e^t(1-t)dt=\frac{1}{2}e-1$.

Câu 24. $I=\int\limits_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sin x\cdot\sqrt{\sin^2 x+\frac{1}{2}}\,dx$ • *Đặt* $t=\cos x$. $I=\frac{3}{16}(\pi+2)$

Câu 25. $I=\int\limits_{0}^{\frac{\pi}{4}} \frac{\sin 4x}{\sqrt{\sin^6 x+\cos^6 x}}dx$

• $I=\int_0^{\frac{\pi}{4}}\frac{\sin 4x}{\sqrt{1-\frac{3}{4}\sin^2 2x}}dx$. Đặt $t=1-\frac{3}{4}\sin^2 2x \Rightarrow \text{I}=\int_1^{\frac{1}{4}}\left(-\frac{2}{3}\frac{1}{\sqrt{t}}\right)dt=\frac{4}{3}\sqrt{t}\Big|_{\frac{1}{4}}^{1}=\frac{2}{3}$.

Câu 26. $I=\int_0^{\frac{\pi}{2}}\frac{\sin x}{\left(\sin x+\sqrt{3}\cos x\right)^3}dx$

• *Ta có:* $\sin x+\sqrt{3}\cos x=2\cos\left(x-\frac{\pi}{6}\right)$;

$\sin x=\sin\left(\left(x-\frac{\pi}{6}\right)+\frac{\pi}{6}\right)=\frac{\sqrt{3}}{2}\sin\left(x-\frac{\pi}{6}\right)+\frac{1}{2}\cos\left(x-\frac{\pi}{6}\right)$

$\Rightarrow I=\frac{\sqrt{3}}{16}\int_0^{\frac{\pi}{2}}\frac{\sin\left(x-\frac{\pi}{6}\right)dx}{\cos^3\left(x-\frac{\pi}{6}\right)}+\frac{1}{16}\int_0^{\frac{\pi}{2}}\frac{dx}{\cos^2\left(x-\frac{\pi}{6}\right)}=\frac{\sqrt{3}}{6}$

Câu 27. $I=\int_{-\frac{\pi}{3}}^{\frac{\pi}{4}}\frac{\sin x\sqrt{1-\cos^2 x}}{\cos^2 x}dx$

• $I=\int_{-\frac{\pi}{3}}^{\frac{\pi}{4}}\frac{\sin x}{\cos^2 x}\sqrt{1-\cos^2 x}.dx=\int_{-\frac{\pi}{3}}^{\frac{\pi}{4}}\frac{\sin x}{\cos^2 x}|\sin x|dx=\int_{-\frac{\pi}{3}}^{0}\frac{\sin x}{\cos^2 x}|\sin x|dx+\int_{-0}^{\frac{\pi}{4}}\frac{\sin x}{\cos^2 x}|\sin x|dx$

$=-\int_{-\frac{\pi}{3}}^{0}\frac{\sin^2 x}{\cos^2 x}dx+\int_0^{\frac{\pi}{4}}\frac{\sin^2 x}{\cos^2 x}dx=\frac{7\pi}{12}-\sqrt{3}-1$.

Câu 28. $I=\int_0^{\frac{\pi}{6}}\frac{1}{\sin x+\sqrt{3}\cos x}dx$

• $I=\int_0^{\frac{\pi}{6}}\frac{1}{\sin x+\sqrt{3}\cos x}dx=\frac{1}{2}\int_0^{\frac{\pi}{6}}\frac{1}{\sin\left(x+\frac{\pi}{3}\right)}dx=\frac{1}{2}\int_0^{\frac{\pi}{6}}\frac{\sin\left(x+\frac{\pi}{3}\right)}{1-\cos^2\left(x+\frac{\pi}{3}\right)}dx$.

Đặt $t=\cos\left(x+\frac{\pi}{3}\right)\Rightarrow dt=-\sin\left(x+\frac{\pi}{3}\right)dx \Rightarrow I=\frac{1}{2}\int_0^{\frac{1}{2}}\frac{1}{1-t^2}dt=\frac{1}{4}\ln 3$

Câu 29. $I=\int_0^{\frac{\pi}{2}}\sqrt{1-\sqrt{3}\sin 2x+2\cos^2 x}dx$

• $I = \int_0^{\frac{\pi}{2}} \left|\sin x - \sqrt{3}\cos x\right| dx = I = \int_0^{\frac{\pi}{3}} \left|\sin x - \sqrt{3}\cos x\right| dx + \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \left|\sin x - \sqrt{3}\cos x\right| dx = 3 - \sqrt{3}$

Câu 30. $I = \int_0^{\frac{\pi}{2}} \frac{\sin x dx}{(\sin x + \cos x)^3}$

• *Đặt* $x = \frac{\pi}{2} - t \Rightarrow dx = -dt \Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\cos t dt}{(\sin t + \cos t)^3} = \int_0^{\frac{\pi}{2}} \frac{\cos x dx}{(\sin x + \cos x)^3}$

$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} \frac{dx}{(\sin x + \cos x)^2} = \frac{1}{2}\int_0^{\frac{\pi}{2}} \frac{dx}{\sin^2(x + \frac{\pi}{4})} = -\frac{1}{2}\cot(x + \frac{\pi}{4})\Big|_0^{\frac{\pi}{4}} = 1 \Rightarrow I = \frac{1}{2}$

Câu 31. $I = \int_0^{\frac{\pi}{2}} \frac{7\sin x - 5\cos x}{(\sin x + \cos x)^3} dx$

• *Xét:* $I_1 = \int_0^{\frac{\pi}{2}} \frac{\sin x dx}{\left(\sin x + \cos x\right)^3}$; $I_2 = \int_0^{\frac{\pi}{2}} \frac{\cos x dx}{\left(\sin x + \cos x\right)^3}$.

Đặt $x = \frac{\pi}{2} - t$. *Ta chứng minh được* $I_1 = I_2$

Tính $I_1 + I_2 = \int_0^{\frac{\pi}{2}} \frac{dx}{\left(\sin x + \cos x\right)^2} = \int_0^{\frac{\pi}{2}} \frac{dx}{2\cos^2(x - \frac{\pi}{4})} = \frac{1}{2}\tan(x - \frac{\pi}{4})\Big|_0^{\frac{\pi}{2}} = 1$

$\Rightarrow I_1 = I_2 = \frac{1}{2} \Rightarrow I = 7I_1 - 5I_2 = 1.$

Câu 32. $I = \int_0^{\frac{\pi}{2}} \frac{3\sin x - 2\cos x}{(\sin x + \cos x)^3} dx$

• *Đặt* $x = \frac{\pi}{2} - t \Rightarrow dx = -dt \Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{3\cos t - 2\sin t}{(\cos t + \sin t)^3} dt = \int_0^{\frac{\pi}{2}} \frac{3\cos x - 2\sin x}{(\cos x + \sin x)^3} dx$

$\Rightarrow 2I = I + I = \int_0^{\frac{\pi}{2}} \frac{3\sin x - 2\cos x}{(\sin x + \cos x)^3} dx + \int_0^{\frac{\pi}{2}} \frac{3\cos x - 2\sin x}{(\cos x + \sin x)^3} dx = \int_0^{\frac{\pi}{2}} \frac{1}{(\sin x + \cos x)^2} dx = 1 \Rightarrow I = \frac{1}{2}.$

Câu 33. $I = \int_0^{\pi} \frac{x\sin x}{1 + \cos^2 x} dx$

• *Đặt* $x = \pi - t \Rightarrow dx = -dt \Rightarrow I = \int_0^{\pi} \frac{(\pi - t)\sin t}{1 + \cos^2 t} dt = \pi\int_0^{\pi} \frac{\sin t}{1 + \cos^2 t} dt - I$

$$\Rightarrow 2I = \pi\int_0^{\pi}\frac{\sin t}{1+\cos^2 t}dt = -\pi\int_0^{\pi}\frac{d(\cos t)}{1+\cos^2 t} = \pi\left(\frac{\pi}{4}+\frac{\pi}{4}\right) \Rightarrow I = \frac{\pi^2}{8}$$

Câu 34. $I = \int_0^{\frac{\pi}{2}}\frac{\cos^4 x\sin x}{\cos^3 x+\sin^3 x}dx$

• *Đặt* $x = \frac{\pi}{2} - t \Rightarrow dx = -dt \Rightarrow I = -\int_{\frac{\pi}{2}}^{0}\frac{\sin^4 t\cos t}{\cos^3 t+\sin^3 t}dt = \int_0^{\frac{\pi}{2}}\frac{\sin^4 x\cos x}{\cos^3 x+\sin^3 x}dx$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}}\frac{\cos^4 x\sin x+\sin^4 x\cos x}{\sin^3 x+\cos^3 x}dx = \int_0^{\frac{\pi}{2}}\frac{\sin x\cos x(\sin^3 x+\cos^3 x)}{\sin^3 x+\cos^3 x}dx = \frac{1}{2}\int_0^{\frac{\pi}{2}}\sin 2x dx = \frac{1}{2}$$

$\Rightarrow I = \frac{1}{4}$.

Câu 35. $I = \int_0^{\frac{\pi}{2}}\left[\frac{1}{\cos^2(\sin x)} - \tan^2(\cos x)\right]dx$

• *Đặt* $x = \frac{\pi}{2} - t \Rightarrow dx = -dt$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}}\left[\frac{1}{\cos^2(\cos t)} - \tan^2(\sin t)\right]dt = \int_0^{\frac{\pi}{2}}\left[\frac{1}{\cos^2(\cos x)} - \tan^2(\sin x)\right]dx$$

Do đó: $2I = \int_0^{\frac{\pi}{2}}\left[\frac{1}{\cos^2(\sin x)} + \frac{1}{\cos^2(\cos x)} - \tan^2(\cos x) - \tan^2(\sin x)\right]dx = 2\int_0^{\frac{\pi}{2}}dt = \pi$

$\Rightarrow I = \frac{\pi}{2}$.

Câu 36. $I = \int_0^{\frac{\pi}{4}}\frac{\cos x - \sin x}{\sqrt{3-\sin 2x}}dx$

• Đặt $u = \sin x + \cos x \Rightarrow I = \int_1^{\sqrt{2}}\frac{du}{\sqrt{4-u^2}}$. Đặt $u = 2\sin t \Rightarrow I = \int_{\frac{\pi}{6}}^{\frac{\pi}{4}}\frac{2\cos t dt}{\sqrt{4-4\sin^2 t}} = \int_{\frac{\pi}{6}}^{\frac{\pi}{4}}dt = \frac{\pi}{12}$.

Câu 37. $I = \int_0^{\frac{\pi}{3}}\frac{\sin x}{\cos x\sqrt{3+\sin^2 x}}dx$

• *Đặt* $t = \sqrt{3+\sin^2 x} = \sqrt{4-\cos^2 x}$. *Ta có:* $\cos^2 x = 4 - t^2$ *và* $dt = \frac{\sin x\cos x}{\sqrt{3+\sin^2 x}}dx$.

$$I = \int_0^{\frac{\pi}{3}}\frac{\sin x}{\cos x\sqrt{3+\sin^2 x}}.dx = \int_0^{\frac{\pi}{3}}\frac{\sin x.\cos x}{\cos^2 x\sqrt{3+\sin^2 x}}dx = \int_{\sqrt{3}}^{\frac{\sqrt{15}}{2}}\frac{dt}{4-t^2} = \frac{1}{4}\int_{\sqrt{3}}^{\frac{\sqrt{15}}{2}}\left(\frac{1}{t+2} - \frac{1}{t-2}\right)dt$$

$$= \frac{1}{4}\ln\left|\frac{t+2}{t-2}\right|\Bigg|_{\sqrt{3}}^{\frac{\sqrt{15}}{2}} = \frac{1}{4}\left(\ln\left|\frac{\sqrt{15}+4}{\sqrt{15}-4}\right| - \ln\left|\frac{\sqrt{3}+2}{\sqrt{3}-2}\right|\right) = \frac{1}{2}\left(\ln\left(\sqrt{15}+4\right)-\ln\left(\sqrt{3}+2\right)\right).$$

Câu 38. $I = \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{x+(x+\sin x)\sin x}{\sin^3 x+\sin^2 x}dx$

• $I = \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{x}{\sin^2 x}dx + \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{dx}{1+\sin x}$.

+ *Tính* $I_1 = \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{x}{\sin^2 x}dx$. *Đặt* $\begin{cases} u = x \\ dv = \dfrac{dx}{\sin^2 x} \end{cases} \Rightarrow \begin{cases} du = dx \\ v = -\cot x \end{cases} \Rightarrow I_1 = \frac{\pi}{\sqrt{3}}$

+ *Tính* $I_2 = \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{dx}{1+\sin x} = \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{dx}{1+\cos\left(\frac{\pi}{2}-x\right)} = \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{dx}{2\cos^2\left(\frac{\pi}{4}-\frac{x}{2}\right)} = 4-2\sqrt{3}$

Vậy: $I = \frac{\pi}{\sqrt{3}} + 4 - 2\sqrt{3}$.

Câu 39. $I = \int_0^{\frac{\pi}{2}} \frac{\sin 2x}{\sqrt{\cos^2 x+4\sin^2 x}}dx$

• $I = \int_0^{\frac{\pi}{2}} \frac{2\sin x\cos x}{\sqrt{3\sin^2 x+1}}dx$. Đặt $u = \sqrt{3\sin^2 x+1} \Rightarrow I = \int_1^2 \frac{\frac{2}{3}udu}{u} = \frac{2}{3}\int_1^2 du = \frac{2}{3}$

Câu 40. $I = \int_0^{\frac{\pi}{6}} \frac{\tan\left(x-\frac{\pi}{4}\right)}{\cos 2x}dx$

• $I = \int_0^{\frac{\pi}{6}} \frac{\tan\left(x-\frac{\pi}{4}\right)}{\cos 2x}dx = -\int_0^{\frac{\pi}{6}} \frac{\tan^2 x+1}{(\tan x+1)^2}dx$. *Đặt* $t = \tan x \Rightarrow dt = \frac{1}{\cos^2 x}dx = (\tan^2 x+1)dx$

$$\Rightarrow I = -\int_0^{\frac{1}{\sqrt{3}}} \frac{dt}{(t+1)^2} = \frac{1}{t+1}\Bigg|_0^{\frac{1}{\sqrt{3}}} = \frac{1-\sqrt{3}}{2}.$$

Câu 41. $I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\cot x}{\sin x.\sin\left(x+\frac{\pi}{4}\right)}dx$

• $I = \sqrt{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\cot x}{\sin^2 x(1+\cot x)}dx$. *Đặt* $1+\cot x = t \Rightarrow \frac{1}{\sin^2 x}dx = -dt$

$$\Rightarrow I = \sqrt{2}\int_{\frac{\sqrt{3}+1}{\sqrt{3}}}^{\sqrt{3}+1} \frac{t-1}{t}dt = \sqrt{2}\left(t-\ln t\right)\Bigg|_{\frac{\sqrt{3}+1}{\sqrt{3}}}^{\sqrt{3}+1} = \sqrt{2}\left(\frac{2}{\sqrt{3}}-\ln\sqrt{3}\right)$$

Câu 42. $I = \int\limits_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{dx}{\sin^2 x.\cos^4 x}$

• *Ta có:* $I = 4.\int\limits_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{dx}{\sin^2 2x.\cos^2 x}$. *Đặt* $t = \tan x \Rightarrow dx = \frac{dt}{1+t^2}$

$\Rightarrow I = \int\limits_{1}^{\sqrt{3}} \frac{(1+t^2)^2 dt}{t^2} = \int\limits_{1}^{\sqrt{3}} (\frac{1}{t^2} + 2 + t^2)dt = (-\frac{1}{t} + 2t + \frac{t^3}{3})\Big|_1^{\sqrt{3}} = \frac{8\sqrt{3}-4}{3}$

Câu 43. $I = \int\limits_{0}^{\frac{\pi}{4}} \frac{\sin x}{5\sin x.\cos^2 x + 2\cos x} dx$

• *Ta có:* $I = \int\limits_{0}^{\frac{\pi}{4}} \frac{\tan x}{5\tan x + 2(1+\tan^2 x)}.\frac{1}{\cos^2 x} dx$. *Đặt* $t = \tan x$,

$\Rightarrow I = \int\limits_{0}^{1} \frac{t}{2t^2 + 5t + 2} dt = \frac{1}{3}\int\limits_{0}^{1} \left(\frac{2}{t+2} - \frac{1}{2t+1}\right) dt = \frac{1}{2}\ln 3 - \frac{2}{3}\ln 2$

Câu 44. $I = \int\limits_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\sin^2 x dx}{\cos^4 x(\tan^2 x - 2\tan x + 5)}$

• *Đặt* $t = \tan x \Rightarrow dx = \frac{dt}{1+t^2}$ $\Rightarrow I = \int\limits_{-1}^{1} \frac{t^2 dt}{t^2 - 2t + 5} = 2 + \ln\frac{2}{3} - 3\int\limits_{-1}^{1} \frac{dt}{t^2 - 2t + 5}$

Tính $I_1 = \int\limits_{-1}^{1} \frac{dt}{t^2 - 2t + 5}$. *Đặt* $\frac{t-1}{2} = \tan u \Rightarrow I_1 = \frac{1}{2}\int\limits_{-\frac{\pi}{4}}^{0} du = \frac{\pi}{8}$. *Vậy* $I = 2 + \ln\frac{2}{3} - \frac{3\pi}{8}$.

Câu 45. $I = \int\limits_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\sin^2 x}{\sin 3x} dx$.

• $I = \int\limits_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\sin^2 x}{3\sin x - 4\sin^3 x} dx = \int\limits_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\sin x}{4\cos^2 x - 1} dx$

Đặt $t = \cos x \Rightarrow dt = -\sin x dx \Rightarrow$ $I = -\int\limits_{\frac{\sqrt{3}}{2}}^{0} \frac{dt}{4t^2 - 1} = \frac{1}{4}\int\limits_{0}^{\frac{\sqrt{3}}{2}} \frac{dt}{t^2 - \frac{1}{4}} = \frac{1}{4}\ln(2-\sqrt{3})$

Câu 46. $I = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\sin x - \cos x}{\sqrt{1+\sin 2x}} dx$

• *Ta có:* $\sqrt{1+\sin 2x} = |\sin x + \cos x| = \sin x + \cos x$ *(vì* $x \in \left[\frac{\pi}{4};\frac{\pi}{2}\right]$*)*

$\Rightarrow I = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\sin x - \cos x}{\sin x + \cos x}dx$. *Đặt* $t = \sin x + \cos x \Rightarrow dt = (\cos x - \sin x)dx$

$\Rightarrow I = \int_1^{\sqrt{2}} \frac{1}{t}dt = \ln|t|\Big|_1^{\sqrt{2}} = \frac{1}{2}\ln 2$

Câu 47. $I = 2\int_1^2 \sqrt[6]{1-\cos^3 x}.\sin x.\cos^5 x dx$

• *Đặt* $t = \sqrt[6]{1-\cos^3 x} \Leftrightarrow t^6 = 1-\cos^3 x \Rightarrow 6t^5 dt = 3\cos^2 x \sin x dx \Rightarrow dx = \frac{2t^5 dt}{\cos^2 x \sin x}$

$\Rightarrow I = 2\int_0^1 t^6(1-t^6)dt = 2\left(\frac{t^7}{7} - \frac{t^{13}}{13}\right)\Bigg|_0^1 = \frac{12}{91}$

Câu 48. $I = \int_0^{\frac{\pi}{4}} \frac{\tan x dx}{\cos x\sqrt{1+\cos^2 x}}$

• *Ta có:* $I = \int_0^{\frac{\pi}{4}} \frac{\tan x dx}{\cos^2 x\sqrt{\tan^2 x + 2}}$. *Đặt* $t = \sqrt{2+\tan^2 x} \Rightarrow t^2 = 2 + \tan^2 x \Rightarrow tdt = \frac{\tan x}{\cos^2 x}dx$

$\Rightarrow I = \int_{\sqrt{2}}^{\sqrt{3}} \frac{tdt}{t} = \int_{\sqrt{2}}^{\sqrt{3}} dt = \sqrt{3} - \sqrt{2}$

Câu 49. $I = \int_0^{\frac{\pi}{2}} \frac{\cos 2x}{(\cos x - \sin x + 3)^3}dx$ • *Đặt* $t = \cos x - \sin x + 3 \Rightarrow I = \int_2^4 \frac{t-3}{t^3}dt = -\frac{1}{32}$.

Câu 50. $I = \int_0^{\frac{\pi}{4}} \frac{\sin 4x}{\cos^2 x.\sqrt{\tan^4 x + 1}}dx$

• *Ta có:* $I = \int_0^{\frac{\pi}{4}} \frac{\sin 4x}{\sqrt{\sin^4 x + \cos^4 x}}dx$. *Đặt* $t = \sqrt{\sin^4 x + \cos^4 x}$ $\Rightarrow I = -2\int_1^{\frac{\sqrt{2}}{2}} dt = 2 - \sqrt{2}$.

Câu 51. $I = \int_0^{\frac{\pi}{4}} \frac{\sin 4x}{1+\cos^2 x}dx$

• *Ta có:* $I = \int_0^{\frac{\pi}{4}} \frac{2\sin 2x(2\cos^2 x - 1)}{1+\cos^2 x}dx$. *Đặt* $t = \cos^2 x \Rightarrow I = -\int_1^{\frac{1}{2}} \frac{2(2t-1)}{t+1}dt = 2 - 6\ln\frac{1}{3}$.

Câu 52. $I = \int_0^{\frac{\pi}{6}} \frac{\tan(x - \frac{\pi}{4})}{\cos 2x}dx$

• *Ta có:* $I = -\int_0^{\frac{\pi}{6}} \frac{\tan^2 x + 1}{(\tan x + 1)^2} dx$. *Đặt* $t = \tan x \Rightarrow I = -\int_0^{\frac{1}{\sqrt{3}}} \frac{dt}{(t+1)^2} = \frac{1-\sqrt{3}}{2}$.

Câu 53. $I = \int_0^{\frac{\pi}{6}} \frac{\tan^3 x}{\cos 2x} dx$

• *Ta có:* $I = \int_0^{\frac{\pi}{6}} \frac{\tan^3 x}{\cos^2 x - \sin^2 x} dx = \int_0^{\frac{\pi}{6}} \frac{\tan^3 x}{\cos^2 x(1 - \tan^2 x)} dx$.

Đặt $t = \tan x \Rightarrow I = \int_0^{\frac{\sqrt{3}}{3}} \frac{t^3}{1-t^2} dt = -\frac{1}{6} - \frac{1}{2}\ln\frac{2}{3}$.

Câu 54. $I = \int_0^{\frac{\pi}{2}} \frac{\cos x}{\sqrt{7 + \cos 2x}} dx$

• $I = \frac{1}{\sqrt{2}} \int_0^{\frac{\pi}{2}} \frac{\cos x\, dx}{\sqrt{2^2 - \sin^2 x}} = \frac{\pi}{6\sqrt{2}}$

Câu 55. $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{dx}{\sqrt[4]{\sin^3 x.\cos^5 x}}$

• *Ta có:* $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{1}{\sqrt[4]{\frac{\sin^3 x}{\cos^3 x}.\cos^8 x}} dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{1}{\sqrt[4]{\tan^3 x}}.\frac{1}{\cos^2 x} dx$.

Đặt $t = \tan x \Rightarrow I = \int_1^{\sqrt{3}} t^{-\frac{3}{4}} dt = 4\left(\sqrt[8]{3} - 1\right)$

Câu 56. $I = \int_0^{\pi} x\left(\frac{\cos^3 x + \cos x + \sin x}{1 + \cos^2 x}\right) dx$

• *Ta có:* $I = \int_0^{\pi} x\left(\frac{\cos x(1+\cos^2 x) + \sin x}{1 + \cos^2 x}\right) dx = \int_0^{\pi} x.\cos x.dx + \int_0^{\pi} \frac{x.\sin x}{1+\cos^2 x} dx = J + K$

+ *Tính* $J = \int_0^{\pi} x.\cos x.dx$. *Đặt* $\begin{cases} u = x \\ dv = \cos x dx \end{cases} \Rightarrow \begin{cases} du = dx \\ v = \sin x \end{cases} \Rightarrow J = -2$

+ *Tính* $K = \int_0^{\pi} \frac{x.\sin x}{1+\cos^2 x} dx$. *Đặt* $x = \pi - t \Rightarrow dx = -dt$

$\Rightarrow K = \int_0^{\pi} \frac{(\pi - t).\sin(\pi - t)}{1 + \cos^2(\pi - t)} dt = \int_0^{\pi} \frac{(\pi - t).\sin t}{1 + \cos^2 t} dt = \int_0^{\pi} \frac{(\pi - x).\sin x}{1+\cos^2 x} dx$

$\Rightarrow 2K = \int_0^{\pi} \frac{(x + \pi - x).\sin x}{1 + \cos^2 x} dx = \pi \int_0^{\pi} \frac{\sin x.dx}{1+\cos^2 x} \Rightarrow K = \frac{\pi}{2}\int_0^{\pi} \frac{\sin x.dx}{1 + \cos^2 x}$

Đặt $t=\cos x \Rightarrow K=\frac{\pi}{2}\int_{-1}^{1}\frac{dt}{1+t^2}$, *đặt* $t=\tan u \Rightarrow dt=(1+\tan^2 u)du$

$$\Rightarrow K=\frac{\pi}{2}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{(1+\tan^2 u)du}{1+\tan^2 u}=\frac{\pi}{2}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}du=\frac{\pi}{2}.u\Big|_{-\frac{\pi}{4}}^{\frac{\pi}{4}}=\frac{\pi^2}{4}$$

Vậy $I=\frac{\pi^2}{4}-2$

Câu 57. $\mathrm{I}=\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\frac{\cos x}{\sin x\sqrt{3+\cos^2 x}}dx$

• *Ta có:* $I=\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\frac{\sin x\cos x}{\sin^2 x\sqrt{3+\cos^2 x}}dx$. *Đặt* $t=\sqrt{3+\cos^2 x}$

$$\Rightarrow I=\int_{\sqrt{3}}^{\frac{\sqrt{15}}{2}}\frac{dt}{4-t^2}=\frac{1}{2}\left(\ln(\sqrt{15}+4)-\ln(\sqrt{3}+2)\right)$$

Dạng 3: Đổi biến số dạng 2

Câu 58. $I=\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\sin x\cdot\sqrt{\sin^2 x+\frac{1}{2}}.dx$

• *Đặt* $\cos x=\sqrt{\frac{3}{2}}\sin t,\ \left(0\le t\le\frac{\pi}{2}\right)\Rightarrow I=\frac{3}{2}\int_0^{\frac{\pi}{4}}\cos^2 t dt=\frac{3}{2}\left(\frac{\pi}{4}+\frac{1}{2}\right)$.

Câu 59. $I=\int_0^{\frac{\pi}{2}}\frac{3\sin x+4\cos x}{3\sin^2 x+4\cos^2 x}dx$

• $I=\int_0^{\frac{\pi}{2}}\frac{3\sin x+4\cos x}{3+\cos^2 x}dx=\int_0^{\frac{\pi}{2}}\frac{3\sin x}{3+\cos^2 x}dx+\int_0^{\frac{\pi}{2}}\frac{4\cos x}{3+\cos^2 x}dx=\int_0^{\frac{\pi}{2}}\frac{3\sin x}{3+\cos^2 x}dx+\int_0^{\frac{\pi}{2}}\frac{4\cos x}{4-\sin^2 x}dx$

\+ *Tính* $I_1=\int_0^{\frac{\pi}{2}}\frac{3\sin x}{3+\cos^2 x}dx$. *Đặt* $t=\cos x\Rightarrow dt=-\sin xdx\Rightarrow I_1=\int_0^1\frac{3dt}{3+t^2}$

Đặt $t=\sqrt{3}\tan u\Rightarrow dt=\sqrt{3}(1+\tan^2 u)du\Rightarrow I_1=\int_0^{\frac{\pi}{6}}\frac{3\sqrt{3}(1+\tan^2 u)du}{3(1+\tan^2 u)}=\frac{\pi\sqrt{3}}{6}$

\+ *Tính* $I_2=\int_0^{\frac{\pi}{2}}\frac{4\cos x}{4-\sin^2 x}dx$. *Đặt* $t_1=\sin x\Rightarrow dt_1=\cos xdx$ $I_2=\int_0^1\frac{4dt_1}{4-t_1^2}dt_1=\ln 3$

Vậy: $I = \frac{\pi\sqrt{3}}{6} + \ln 3$

Câu 60. $I = \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{\tan x}{\cos x\sqrt{1+\cos^2 x}} dx$

• *Ta có:* $I = \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{\tan x}{\cos^2 x\sqrt{\frac{1}{\cos^2 x}+1}} dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{\tan x}{\cos^2 x\sqrt{\tan^2 x+2}} dx$

Đặt $u = \tan x \Rightarrow du = \frac{1}{\cos^2 x} dx \Rightarrow I = \int_{\frac{1}{\sqrt{3}}}^{1} \frac{u}{\sqrt{u^2+2}} dx$. *Đặt* $t = \sqrt{u^2+2} \Rightarrow dt = \frac{u}{\sqrt{u^2+2}} du$.

$\Rightarrow I = \int_{\sqrt{\frac{7}{3}}}^{\sqrt{3}} dt = t\Big|_{\sqrt{\frac{7}{3}}}^{\sqrt{3}} = \sqrt{3} - \frac{\sqrt{7}}{\sqrt{3}} = \frac{3-\sqrt{7}}{\sqrt{3}}$.

Câu 61. $I = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\sin\left(x+\frac{\pi}{4}\right)}{2\sin x\cos x - 3} dx$

• *Ta có:* $I = -\frac{1}{\sqrt{2}} \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\sin x + \cos x}{\left(\sin x - \cos x\right)^2 + 2} dx$. *Đặt* $t = \sin x - \cos x \Rightarrow I = -\frac{1}{\sqrt{2}} \int_0^1 \frac{1}{t^2+2} dt$

Đặt $t = \sqrt{2}\tan u \Rightarrow I = -\frac{1}{\sqrt{2}} \int_0^{\arctan\frac{1}{\sqrt{2}}} \frac{\sqrt{2}(1+\tan^2 u)}{2\tan^2 u + 2} du = -\frac{1}{2}\arctan\frac{1}{\sqrt{2}}$

Dạng 4: Tích phân từng phần

Câu 62. $I=\int_{\frac{-\pi}{3}}^{\frac{\pi}{3}}\frac{x\sin x}{\cos^2 x}dx$.

• *Sử dụng công thức tích phân từng phần ta có:*

$$I=\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}}xd\left(\frac{1}{\cos x}\right)=\left.\frac{x}{\cos x}\right|_{-\frac{\pi}{3}}^{\frac{\pi}{3}}-\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}}\frac{dx}{\cos x}=\frac{4\pi}{3}-J,\ \text{với}\ J=\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}}\frac{dx}{\cos x}$$

Để tính J ta đặt $t=\sin x$. *Khi đó* $J=\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}}\frac{dx}{\cos x}=\int_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}}\frac{dt}{1-t^2}=-\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right|\Bigg|_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}}=-\ln\frac{2-\sqrt{3}}{2+\sqrt{3}}$

Vậy $I=\frac{4\pi}{3}-\ln\frac{2-\sqrt{3}}{2+\sqrt{3}}$.

Câu 63. $I=\int_{0}^{\frac{\pi}{2}}\left(\frac{1+\sin x}{1+\cos x}\right).e^x dx$

• *Ta có:* $\frac{1+\sin x}{1+\cos x}=\frac{1+2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}=\frac{1}{2\cos^2\frac{x}{2}}+\tan\frac{x}{2}$

$$\Rightarrow I=\int_{0}^{\frac{\pi}{2}}\frac{e^x dx}{2\cos^2\frac{x}{2}}+\int_{0}^{\frac{\pi}{2}}e^x\tan\frac{x}{2}dx\ =\ e^{\frac{\pi}{2}}$$

Câu 64. $I=\int_{0}^{\frac{\pi}{4}}\frac{x\cos 2x}{(1+\sin 2x)^2}dx$

• *Đặt* $\begin{cases}u=x\\ dv=\frac{\cos 2x}{(1+\sin 2x)^2}dx\end{cases}\Rightarrow\begin{cases}du=dx\\ v=-\frac{1}{1+\sin 2x}\end{cases}$

$$\Rightarrow I=x.\left(-\frac{1}{2}.\frac{1}{1+\sin 2x}\right)\Bigg|_{0}^{\frac{\pi}{4}}+\frac{1}{2}\int_{0}^{\frac{\pi}{4}}\frac{1}{1+\sin 2x}dx=-\frac{\pi}{16}+\frac{1}{2}\int_{0}^{\frac{\pi}{4}}\frac{1}{\sqrt{2}}.\frac{1}{\cos^2\left(x-\frac{\pi}{4}\right)}dx$$

$$=-\frac{\pi}{16}+\frac{1}{2}.\frac{1}{\sqrt{2}}\tan\left(x-\frac{\pi}{4}\right)\Bigg|_{0}^{\frac{\pi}{4}}=-\frac{\pi}{16}+\frac{1}{2}.\frac{\sqrt{2}}{2}(0+1)=\frac{\sqrt{2}}{4}-\frac{\pi}{16}$$